AN ANALYSIS OF THE HUMAN TRANSFER FUNCTION IN AIRCRAFT LONGITUDINAL MOTION

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SUMMARY

This investigation was carried out in an effort to express in analytical terms the relationship between a pilot's input and output while flying a Ryan Navion Aircraft during the performance of a specified maneuver in the longitudinal plane. The maneuver chosen for the pilot to perform was a simple air speed change in which the pilot was required to reduce the aircraft velocity from a given value to a lower assigned value using only the elevator. He was further required to do this in a smooth and expeditious manner and to arrive at his new air speed without overshoot or undershoot. The mathematical relationship may be expressed as the ratio of his output, (the manner in which he positions the elevator), in response to his input (the desire to change air speed a given number of miles per hour). This ratio, $H = \frac{\delta}{\varepsilon}$, is called the human transfer function.

In order to determine the human transfer function, it was necessary to divide the investigation into three parts.

In Part I, an expression for the ratio of the aircraft velocity response to an elevator forcing function in the form of a step was obtained. This ratio, $A = \frac{u}{\delta} = \frac{\varepsilon}{\delta}$, can be considered the aircraft transfer function, and can be expressed with good accuracy by a second order differential equation whose characteristic equation yields roots from

which the period and time to damp to half amplitude of the aircraft phugoid mode may be calculated.

In Part II, several mathematical expressions for the human transfer function were obtained. In arriving at all of the expressions for, $H = \frac{\delta}{\varepsilon}$, certain simplifying assumptions were made. It was considered that the pilot was merely a "black box" in the system, and that his inputs were limited to a visual realization of the error (air speed change desired) and simple functions of the error. No attempt was made to consider any time delay in the pilot's response, nor were any neuro-muscular effects taken into account. A differential equation containing terms of δ, $\dot{\delta}$, $\ddot{\delta}$, ε, $\dot{\varepsilon}$ and $\ddot{\varepsilon}$, and an integro-differential equation containing terms of δ, $\dot{\delta}$, ε, $\dot{\varepsilon}$ and $\int \varepsilon \, dt$ were each solved simultaneously with the aircraft transfer function on an analog computer and time histories of velocity and elevator response were recorded. Comparison of these responses with those taken in flight showed that the differential equation gave slightly better correlation with the velocity response than did the integro-differential equation. However, this difference was slight. Comparison of the computer solution of the elevator response with that recorded in the airplane showed that the integro-differential equation gave the better correlation. It is felt that the integro-differential equation represents the pilot's transfer function reasonably and well. Recommendations were made to check the validity of both expressions

under varying conditions.

In Part III of the investigation, a mock-up simulating the elevator controls of the aircraft was constructed. Fore and aft movement of the control column caused a positive or negative voltage output from the mock-up which corresponded to the elevator angle displacement. This voltage was used as a forcing function on the simplified equations of aircraft motion which were set up on the analog computer. The velocity output on the computer was displayed on a meter located in front of the pilot. An attempt was made to correlate the pilot's movement of the elevator in accomplishing speed changes on the computer with that occurring in the air and with the results obtained in Part II. No satisfactory comparison could be made because of the rough design of the mock-up. It is felt that with an improved mock-up design and with certain recommended changes in the test procedure a fruitful comparison might be made.

SYMBOLS AND SIGN CONVENTION

δ_e Absolute elevator angle in degrees. Positive down.

δ Perturbation elevator angle in degrees taken from an arbitrary reference. Down elevator positive.

$\dot{\delta}, \ddot{\delta}$ $d\delta/dt$ and $d^2\delta/dt^2$ respectively.

ε The error in mph from a desired airspeed reference. Positive error if the speed is higher than desired.

$\dot{\varepsilon}, \ddot{\varepsilon}$ $d\varepsilon/dt$ and $d^2\varepsilon/dt^2$ respectively.

P Aircraft period in seconds.

$T_{1/2}$ Time to damp to half amplitude in seconds.

D The operator d( )/dt.

u Non dimensional velocity change change, $\Delta V/V$.

V Velocity in fps or mph as indicated. Subcripts "i" and "f" are initial and final velocities respectively.

F_s Stick force in pounds.

$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$ Partial derivative of the moment coefficient with respect to angle of attack.

C_{L_α} Slope of the lift curve.

$C_{md\theta} = \frac{\partial C_m}{\partial d\theta}$ Partial derivative of the moment coefficient with respect to $\tau\dot{\theta}$.

$C_{D_\alpha} = \frac{\partial C_D}{\partial \alpha}$ Partial derivative of the drag coefficient with respect to angle of attack.

$C_{md\alpha} = \frac{\partial C_m}{\partial d\alpha}$ Partial derivative of the moment coefficient with respect to $\tau\dot{\alpha}$.

C_{m_δ} Elevator power.

α Angle of attack.

$d\epsilon/d\alpha$ Rate of change of downwash angle with angle of attack.

m Mass of the airplane in slugs.

$\mu = m/\rho Sc$

$\tau = m/\rho SV$

I_y Moment of inertia about the Y-axis in slug ft^2.

$h = 2k_y^2/\mu c^2$

$A = (\frac{\epsilon}{\delta})_{aircraft}$ = Aircraft Transfer Function

$H = (\frac{\delta}{\epsilon})_{human}$ = Human Transfer Function

NAVION SPECIFICATIONS AND NOTATIONS

Wing area, $S = 184.2\ ft^2$

Horizontal tail area, $S_t = 43\ ft^2$

Elevator area = $15.04\ ft^2$

Wing aspect ratio, A, = 6.04

Wing span, b, = 33.38 ft.

Tail span, b_t = 13.17 ft.

Wing MAC, c, = 5.7 ft.

Tail MAC, c_t, = 3.34 ft.

Tail length, l_t = 15.04 ft.

Incidence, wing, i_w, = 2^o (at root), 1^o (tip)

Incidence, Tail, i_t, = -3^o

Airfoil, Wing, Root: NACA 4415 R, Tip: NACA 6410 R

Airfoil, Tail, NACA 0010

Dihedral, 7.5^o

Wing Taper ratio, λ, = .54

Wing aerodynamic center, a.c., .242

Elevator:

Root chord = 1.5 ft.

Tip chord = 1.0 ft.

Stabilizer:

Root chord = 2.5 ft.

Tip chord = 1.67 ft.

Thrust axis is parallel to fuselage reference line.

C_L = lift coefficient. Subscripts a, w, t denote airplane, wing and tail, respectively

C_m = moment coefficient

V = tail volume = $\frac{S_t l_t}{S_w c}$ = .616

c.g. = center of gravity

α = angle of attack. Subcripts w, t, and p denote wing, tail and propeller thrust axis respectively.

X_{cg} = c.g. position in percent of MAC

X_{ac} = a.c. position in percent of MAC

S_p = propeller disk area = 38.5 ft^2

N_o = control-fixed neutral point.

INTRODUCTION

During the past ten to fifteen years the advancements in the engineering and other physical sciences have far surpassed those of the biological sciences. As a result, new equipment of every sort is being developed, built, and put into use with no scientific assurance that, from the standpoint of the human operator, it is completely useful or habitable. As a consequence, the field of human engineering with its task of increasing the effectiveness of a man-machine system by treating it as a unified system has become of increasing importance. Organizations in various universities, colleges, civilian enterprise, and the military are attempting to apply the known principles of psychology, physiology, anthropology, and medicine, as well as engineering techniques, to the man-machine relationship and equipment design. Further, they have set up elaborate programs for research in order to increase their knowledge in these fields.

The problem of aircraft control illustrates the complexity of the human response. In controlling the aircraft, the pilot receives certain data from his flight instruments which he judges, compares, accepts or rejects, and then moves his controls accordingly. This control movement changes the information sent to the pilot by his instruments, and the pilot must continually re-evaluate this information in an effort to make the aircraft perform

in the manner which he desires. In actuality, this is more than a simple, error-sensing, closed loop servo system, for the pilot is capable of fulfilling (and does) the additional functions of examining the information received from his instruments in the light of their rates of change and their time duration. This means that he is acting in some respects as a computer which senses errors and their derivatives or integrals. Theoretically it should be possible to express the pilots relationship between the inputs and outputs of the system (i.e. the error in some instrument from a desired value and the pilot's control movement to correct the instrument reading) as some mathematical relationship corresponding to a closed loop servo system which includes one or more types of feedback.

A mathematical expression of this sort is tantamount to considering the pilot as a "black box" in the system, and does not consider the biological, psychological, or physiological principles which enable him to perform these functions. However, such a mathematical expression should be of interest in attempting to explain why the human acts as he does.

The authors of this report have elected to use the "black box" approach in their investigation of the relationship of the human and the aircraft. One simple task was assigned to the pilot. He was required to decrease his air speed from a given value to another specified velocity,

smoothly and without overshoot.

In an effort to express the pilot's actions in performing this task by an analytical expression, or transfer function, the problem was divided into three parts.

I Determination of the aircraft transfer function.

II Determination of the human transfer function.

III Check of the validity of the human transfer function by a Mock-Up.

PART I

DETERMINATION OF THE AIRCRAFT TRANSFER FUNCTION

THEORY

In Refs. 1 and 2 it is shown that the aircraft equations of motion can be combined to give a characteristic equation of the form:

$$C_4\lambda^4 + C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0 = 0 \tag{1}$$

Solution of this equation (confirmed by flight experience) has shown that the aircraft has two modes of longitudinal motion. The large pair of roots correspond to a short period, heavily damped mode, while the small pair of roots correspond to a lightly damped, long period oscillation called the phugoid. The response of an aircraft of normal configuration to an elevator forcing function is such that the short period mode is so heavily damped that either the pilot is not aware of its presence, or, if aware, is unable to make a correction before the motion is completely damped out. This is particularly true of the aircraft velocity response which, in general, experiences no visible change. However, the presence of the phugoid with its long period (30 to 60 seconds) and light damping (time to half amplitude of 30 seconds or longer) is easily detected by the pilot as a change of airspeed and as a change of attitude.

Thus, as far as the pilot is concerned, the important part of the aircraft velocity response to an elevator

forcing function is the phugoid mode. The characteristic equation, when the short period mode is neglected, then reduces to the form:

$$K_2 \lambda^2 + K_1 \lambda + K_0 = 0 \tag{2}$$

If we neglect the short period mode and assume that the aircraft has a transfer function of the form:

$$\frac{u}{\delta} = \frac{1}{(aD^2 + bD + K)} \tag{3}$$

where $u = \frac{\Delta V}{V}$ $\qquad \delta$ = elevator angle applied, degrees

$K = \delta_{ss} / u_{ss}$

$D = \frac{d}{dt}$ $\qquad V$ = initial velocity, mph

Then the characteristic equation would be:

$$a\lambda^2 + b\lambda + K = 0 \tag{4}$$

which is of the same form as Equation (2).

The roots of Equation (4) would be:

$$\lambda_{1,2} = -\frac{b}{2a} \pm \sqrt{-\left(\frac{b}{2a}\right)^2 + \frac{K}{a}}\; i \tag{5}$$

If the aircraft were very lightly damped, then,

$$\frac{b}{2a} \ll \frac{K}{a}$$

and the equation would reduce to:

$$\lambda_{1,2} = -\frac{b}{2a} \pm \sqrt{\frac{K}{a}}\; i \tag{6}$$

The expressions appearing in Equation (6) are related to the period and time to damp to one half amplitude by the following equations (Ref. 1):

$$P = \frac{2\pi}{\omega} = 2\pi \Big/ \sqrt{K/a} \qquad \text{Sec.} \tag{7}$$

$$T_{1/2} = .693 \Big/ \frac{b}{2a} \qquad \text{Sec.} \tag{8}$$

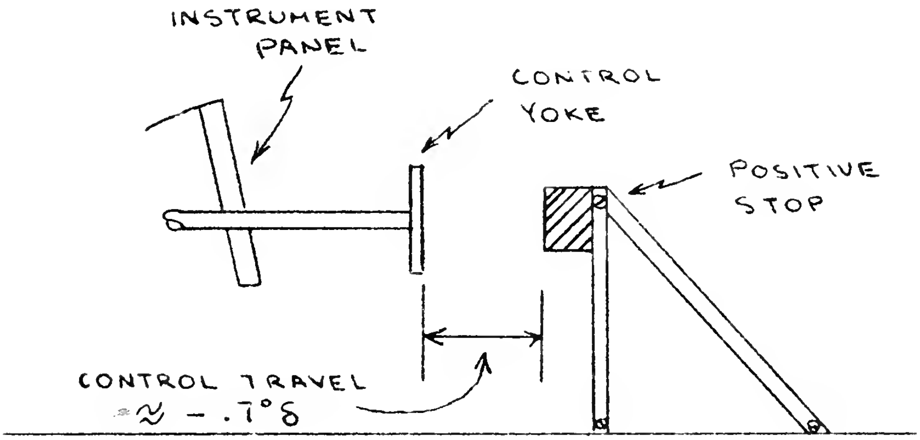
If a known elevator forcing function were applied to the aircraft in flight, and the time history of the aircraft velocity could be recorded, then the period and the time to damp to one half amplitude could be measured from the time history of the velocity. From this data, and knowing that

$$K = \frac{\delta \text{ steady state or final}}{u \text{ steady state or final}}$$

the value of the unknown coefficients of the aircraft transfer function, Equation (3), could be computed from Equations (7) and (8).

PROCEDURE AND DISCUSSION

As indicated in Appendix A, the aircraft was instrumented to record velocity and elevator angle. In order for the pilot (seated at the left hand controls) to apply an elevator step function to the aircraft in flight, a positive stop to control movement was erected on the right hand controls. A sketch of this stop is shown below.



The stop was adjusted so that the stick movement corresponded roughly to an elevator change of -.7 degrees (up elevator). This elevator deflection produced an airspeed change of approximately 20 miles per hour.

The test flights were conducted in the following manner. All instrumentation was calibrated prior to take off, and upon return. Flights were made during the very early morning in order to have smooth air conditions. Each run was commenced from a base pressure altitude of

4800 feet in order to give an average pressure altitude of 5000 feet during the run. Temperature was recorded at 5000 feet in order to correct the recorded calibrated air speeds to true air speeds. The pilot established a base calibrated air speed of approximately 120 mph at 4800 feet and trimmed the aircraft. After the recording mechanism was started and checked, the pilot moved the yoke sharply rearward until it met the positive stop, and then held it firmly against the stop until all aircraft oscillation had ceased and the aircraft was again in steady flight at a lower airspeed and a less positive elevator angle.

A series of eight runs of this nature were made and the time histories of the aircraft velocity response were recorded. As no effort was made to start each run from exactly the same air speed, and as the step function could not be reproduced exactly from run to run, the aircraft did not settle down to the same speed on each run.

Examination of the time histories of the elevator response showed that there was some random variation of the period and time to damp to half amplitude from run to run. However, all Periods were 36 seconds $\pm$ 1 second, and all Times to half amplitude were 25 seconds $\pm$ 1 second. The δ/u

where $$\delta = \delta_{initial} - \delta_{final}$$

$$u = \frac{V_{initial} - V_{final}}{V_{initial}}$$

was computed for each run and all results lay in the range

$4.70 \pm .10$. Values of P, $T_{\frac{1}{2}}$, and δ/u for the eight runs are tabulated in Table I. Two typical time histories of the velocity response taken from the recording oscillograph record are plotted as Figs. 2 and 3 to show the phugoid mode.

After examination of the results in Table I, it was elected to choose the values obtained in Run 2172 for computing the aircraft transfer function. These values are reproduced below for convenience:

$$V_i = 132 \text{ mph} \qquad u = \frac{V}{V_i} = 22.5/132 = .1705$$

$$V_f = 108.5 \text{ mph} \qquad \frac{\delta}{u} = K = 4.70$$

$$\Delta V = 22.5 \text{ mph} \qquad P = 36 \text{ seconds}$$

$$\delta = .80 \text{ degrees} \qquad T_{\frac{1}{2}} = 25 \text{ seconds}$$

The constants of Equation (3) were then computed from Equations (7) and (8).

$$P = 2\pi/\sqrt{\frac{K}{a}}$$

$$a = \frac{P^2 K}{4\pi^2} = \frac{(36)^2 (4.70)}{4\pi^2} = 153.6$$

$$T_{1/2} = \frac{.693}{b/2a}$$

$$b = \frac{2a(.693)}{T_{1/2}} = \frac{2(153.6)(.693)}{25} = 8.50$$

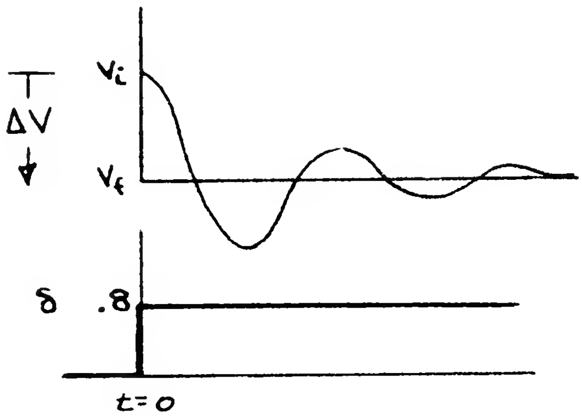
The equation (3) could be written:

$$\frac{u}{\delta} = \frac{1}{(153.6D^2 + 8.50D + 4.70)} \qquad (9)$$

Expressing the transfer function in terms of ΔV, we have:

$$\frac{\Delta V}{\delta} = \frac{1}{(1.164 D^2 + .0644 D + .0355)} \qquad (10)$$

It is seen that the above equation satisfies the initial and final physical conditions:

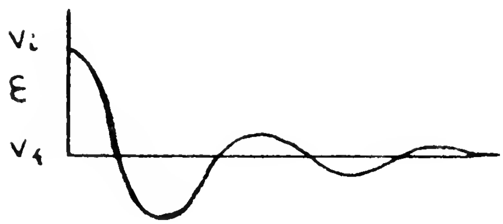


at $t = 0$
$\Delta V = 0$; $\delta = 0$

at $t =$ steady state
$\Delta V =$ 22.5 mph
$\delta = .8°$

Now if we consider that when flying at the higher airspeed, 130 mph, and at an elevator angle of .80 degrees, we desire to decrease the airspeed to 108.5 mph by the application of a step function elevator input, then the elevator angle must be decreased to zero by the step function. Thus if we define

$$\text{error} = \varepsilon = V_i - V_f$$
$$\text{elevator input} = \delta = \delta - \delta_f$$



the aircraft transfer function may be written

$$A = \frac{\varepsilon}{\delta} = \frac{1}{(1.164\,D^2 + .0644\,D + .0355)} \qquad (11)$$

where ε represents the difference between any speed and the speed desired

δ represents the elevator step function required to attain the desired speed.

This is the form in which the aircraft transfer function is used during the remainder of this report.

This expression was then checked by means of an analog computer. The velocity time history was recorded on a Brush Recorder in response to an elevator step function voltage corresponding to .80 degrees. The period and time to half amplitude were found to be 36 seconds and 25 seconds, respectively.

RESULTS

Flight tests indicated that the phugoid mode was relatively constant with a period of 36 seconds and a time to half amplitude of 25 seconds. Further, they show that the ratio of the elevator step input to the speed change resulting from that input is constant.

The aircraft transfer function as determined by flight tests and checked by an analog computer may be expressed as:

$$A = \frac{\epsilon}{\delta} = \frac{1}{(1.164 D^2 + .0644 D + .0355)} \qquad (11)$$

CONCLUSIONS

The pilot in flying the aircraft does not recognize the presence of the short period mode in the velocity response of the aircraft to an elevator input. The recorded time history of the velocity response does not show the short period mode.

The pilot is aware of the phugoid mode caused by an elevator disturbance.

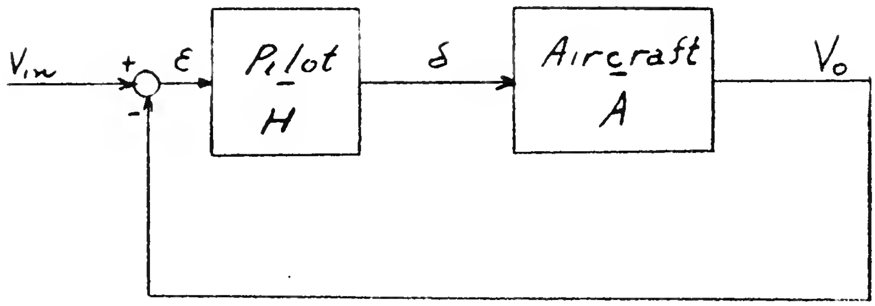
The transfer function of the velocity of the aircraft in response to an elevator forcing function in the form of a step can be closely approximated by a second order differential equation.

PART II

DETERMINATION OF THE PILOT'S TRANSFER FUNCTION

THEORY

In an effort to express the pilot's actions to a desired change in air speed in terms of an output and an input, he may be considered part of a servomechanism. This system is a simple closed loop system, sketched below:



V_{in} is a step function input of velocity (or the desired velocity); V_o is the actual aircraft velocity; the error, ε, is the difference of the two; and δ is the increment of elevator angle from an arbitrary reference.

It was the desire of the authors of this investigation to obtain a mathematical relationship of the pilot's output (δ) versus the input (ε). This mathematical relationship, δ/ε, or transfer function, was desired in the simplest form that would give reasonably accurate results for a smooth air speed change.

The authors felt that it would be possible to express the pilot's actions in a very simple linear relationship, $\delta = a\varepsilon + b\dot{\varepsilon}$; as the pilot sees the error and derivative

of the error merely by watching his air speed indicator reading and rate of change of reading. With practice, the pilot knows approximately where to position his elevator for a desired air speed change, and he positions it accordingly.

Futhermore, there exists the pilot's integrating ability, which physically corresponds to the pilot's desire to change air speed in a reasonable length of time. This ability is also a factor which might effect the pilot's actions, and as such, should be considered.

A number of basic assumptions were made in order to facilitate arriving at an expressable result. It was assumed that the pilot had only visual realization of the error or any simple function of the error, such as the rate of change, accelération or integral of the error. No account was taken of stick force effect, as it was extremely small in the test aircraft; however, it is felt thatthis effect definitely influences pilot reation in extreme speed changes. The pilot's time delay was recognized to be of the order of one-third to three-fourths of a second. It is a variable quantity depending on the task that the pilot happens to be performing, and as such is extremely difficult to evaluate. No means were at the authors' disposal to record time delay; hence for the purpose of this investigation, it was ignored. Throughout

this investigation, the pilot was considered simply a component part of a servomechanism, or a "black box", and physiological relationships such as nerve response were not considered.

PROCEDURE AND DISCUSSION

The flight procedure for this phase of the investigation was identical to that of the aircraft transfer function determination, with the important exception that the aircraft speed was changed from approximately 130 mph true to 110 mph true through the exclusive use of the elevator - power settings remaining constant. This speed change was made in a smooth, expeditious manner. Time histories of calibrated air speed and elevator angle were recorded for over 20 runs, two of which are indicated in Figs. 10 and 11.

The speed was changed on all runs in smooth air conditions at 5,000 feet mean pressure altitude. The pilot practiced before each series of recorded runs in order to make smooth speed changes void of overshoots or undershoots; hence the changes were not of an entirely random nature.

The initial effort to obtain a mathematical relationship between δ and $\mathcal{E}$ was a numerical analysis of $\delta, \mathcal{E}$ and $\dot{\mathcal{E}}$ in an effort to fit a curve of the order $\delta = a\mathcal{E} + b\dot{\mathcal{E}}$. The constants "a" and "b" were determined as indicated in the Sample Calculations. This equation necessitated assuming δ to go immediately to a finite value at $t = 0$. The initial error was considered to be 20, and the final elevator angle and error was zero.

These results were considered poor; so an electronic analog computer means, as illustrated in Fig. 5 was devised

for solving a second order differential equation in δ and ε

$$(aD^2 + bD + 1)\delta = (a'D^2 + b'D + c')\varepsilon$$

simultaneously with the aircraft equation as determined in Part I by varying the coefficients of the human equation to optimize each coefficient and arrive at a near perfect curve fit of δ and ε for a reference air speed change of the test aircraft.

With run 2170 as a reference, these unknown coefficients were varied by adjusting the proper potentiometer settings and their input resistances until the optimum curve fits in δ and ε for aircraft and computer curves were obtained. An original setting was made on each variable potentiometer to make both error and elevator curves stable. Following this, each variable potentiometer was adjusted until it gave optimum results; then the same procedure was repeated. The effect of varying each unknown constant was noted, and this appreciably assisted in obtaining a reasonably rapid curve agreement. The final result was checked with run 2167. The constants "a" and "a'" were quite small and had little effect on improving the curves.

In the aforementioned computer set-up, the original elevator angle and error were assumed to be non-zero and the final steady-state values zero. Initial conditions of +20 mph error and $+.65^{\circ}$ elevator angle were set into the computer problem.

The optimized transfer function obtained was,

$$\frac{\delta}{\varepsilon} = H_1 = - \frac{(.01D^2 + .355D + .042)}{(.01D^2 + 3.15D + 1)}$$

and ignoring the second derivative terms, as their effect is negligible, yields,

$$\frac{\delta}{\varepsilon} = H_1 = - \frac{(.355D + .042)}{(3.15D + 1)}$$

The characteristic equation for the simultaneous equations, H_1 and A, was investigated and found to be,

$$3.67D^3 + 1.389D^2 + .5313D + .0775 = 0$$

the roots of which are,

$$\lambda_1 = - .195; \qquad \lambda_{2,3} = - .0921 \pm .3171$$

An attempt was made to analyze this result, but as the roots are of the same order of magnitude, a thorough analysis in terms of the constants of the transfer function was impossible. It is indicated, however, that the pilot in changing air speed is modifying the phugoid mode by adding a comparatively highly damped convergent mode and increasing the phugoid's damping and frequency.

The possibility of the pilot's integrating ability influencing the transfer function was next considered. This gives an equation of the form,

$$a\dot{\delta} + \delta = a'\dot{\varepsilon} + b'\varepsilon + c'\int \varepsilon \, dt$$

The analog schematic for this determination is shown in Fig. 12 with the final optimized potentiometer settings indicated in Table II. The optimized equation obtained was,

$$\frac{\delta}{\mathcal{E}} = H_2 = - \frac{(.436D + .0703 + \frac{.00522}{D})}{(1.602D + 1)}$$

The characteristic equation for the simultaneous equations, H_2 and A, was investigated and found to be, .

$$D^4 + .678D^3 + .2981D^2 + .0566D + .00279 = 0$$

the roots of which are,

$$\lambda_1 = - .225; \lambda_2 = - .074; \quad \lambda_{3,4} = - .188 \pm .354i$$

These roots indicate modes similar to the second order result, but contain an additional lightly damped convergent mode arising from the presence of the integral term.

In determining the aircraft transfer function for the integral solution, an initial velocity of +20 mph and a δ of zero were assumed. This gave an equation of the form,

$$\delta + a\mathcal{E} + b\dot{\mathcal{E}} + c\ddot{\mathcal{E}} = K$$

For the error and its derivative to go to zero, the steady

state perturbation elevator angle must equal K, or $\delta_{ss} = K$. A value of $\delta = -.65^\circ$ (δ for 20 mph change) was therefore set into the summation point for the aircraft transfer function. This in no way affected the dynamics or solution to the problem, as it merely shifted the arbitrary zero for δ.

RESULTS

In analyzing the results of this phase of the investigation, note should be taken of the physical aspects of the results obtained. The optimized second order differential obtained was:

$$(.01D^2 + 3.15D + 1)\delta = -(01D^2 + .355D + .042)\mathcal{E}$$

This result indicates that the inertia effect of the elevator is small and can safely be neglected, and that even though the pilot is able to detect error acceleration, he changes his speed primarily according to his error and error derivative determination. The necessity of the inclusion of the $\dot{\delta}$ term is indicated due to physical limitations of the pilot to abruptly move his elevator control and his reluctance to move it too rapidly in consideration of aircraft stresses and pilot-passenger comfort. The computer and aircraft reference error curves indicated in Figs. 6 and 7 are excellent, but the elevator curves indicated in Figs. 8 and 9 do not agree too closely. They are within .05 degree and 1 second, however.

By including the pilot's integrating ability the integro-differential equation obtained after optimizing the constants was:

$$(1.602D + 1)\delta = -(.436D + .0\text{[illegible]}3 + \frac{1}{D}x.00522)$$

where "1/D" is the integral. The error curves of computer and aircraft reference were good, having a maximum variation of .3 mph, and the elevator curves were excellent, favorably comparing in both magnitude and phase. These curves are shown in Figs. 13 and 14.

The algebraic sign of all terms of both the second order result and the integro-differential result should be considered. The error term in both human equations indicates to the pilot the proper direction to move his elevator (positive error, negative elevator angle). The error derivative acts as a limiter and tells the pilot he is approaching his desired air speed rapidly (for a high derivative), and therefore to decrease his elevator angle (negative $\dot{\mathcal{E}}$, positive elevator angle. $\dot{\mathcal{E}}$ is always negative for a decrease in air speed). A positive integral of the error tells the pilot not to linger in changing his speed, and also indicates the pilot's ability to detect how long the error has existed (positive integral, negative elevator). It is felt by the authors that the pilot not only has the ability to integrate but also the ability to reevaluate this integral from time to time.

CONCLUSIONS AND RECOMMENDATIONS

In arriving at an expressable pilot's transfer function, three attempts were made to obtain a result in the form of a differential or integro-differential equation. The first, or simple system gave poor results and was not considered. The second order system results were good, and it must be concluded from these results that the second derivative terms are negligible. The integro-differential result was excellent and reasonable, and it is concluded that with further study and investigation, the pilot's integrating ability can further be explained.

It is the authors' belief that the results of this investigation are quite limited in scope, as they were obtained for one pilot changing speed in one aircraft a definite increment under ideal conditions. In considering a servomechanism system, however, and neglecting nonlinearities, test runs in different aircraft, with <u>different</u> aircraft transfer functions, should give close results to those obtained in this report for comparable air speed changes.

A more complete investigation should include a frequency response or impulse integration analysis of various, random air speed changes, as outlined in Chapter 11 of Ref. 8, to verify the pilot's response in terms of only an air speed error, its derivative and its integral.

SAMPLE CALCULATIONS

In order to fit a curve of the form,

$$\delta = a\varepsilon + b\dot{\varepsilon}$$

two simultaneous equations were formed from two sets of data points. Using known values of δ, ε, and $\dot{\varepsilon}$ one equation was formed from an averaging of the sum of even numbered seconds, and another from averaging odd numbered seconds. These two equations in "a" and "b" were then solved simultaneously for a and b as indicated.

Time	δ	=	$a\varepsilon$	+	$b\dot{\varepsilon}$
4	0	=	15.4a	-	2.34b
6	+.45	=	10.3a	-	2.73b
8	+.55	=	5.4a	-	2.21b
10	+.52	=	1.7a	-	1.08b
12	+.37	=	.2a	-	.361b
	+1.89	=	33.0a	-	8.721b

Dividing by five (5),

$$+.378 = 6.6000a - 1.7442b$$

3	-.23	=	17.5a	-	1.91b
5	+.25	=	13.1a	-	2.73b
7	+.53	=	7.6a	-	2.37b
9	+.55	=	3.2a	-	1.75b
11	+.48	=	.9a	-	.716b
	+1.58	=	42.3a	-	9.476b

or, $+.316 = 8.460a - 1.89520b$

Determinant of coefficients, D, = $\begin{vmatrix} 6.60 & -1.7442 \\ 8.46 & -1.8952 \end{vmatrix}$

$D = + 2.24761$

$D_a = - .1652184, \quad a = - .0735085$

$D_b = -1.11228, \quad b = - .494872$

Therefore,

$$\delta = -.0735085\,\mathcal{E} \; - .494872\,\dot{\mathcal{E}}$$

PART III

CONSTRUCTION AND TESTING OF AN ELEVATOR CONTROL SIMULATOR

THEORY

The equations of aircraft longitudinal motion used in this section of the report are the simplified equations of Lift, Drag and Moment developed in Chapter 10 of Ref. 1. These non-dimensional equations are:

$$\underline{\text{LIFT}}: \quad \frac{C_L}{\tau}u + \left(\frac{C_{L\alpha}}{2\tau} + d\right)\alpha - d\theta = 0$$

$$\underline{\text{DRAG}}: \quad \left(\frac{C_D}{\tau} + d\right)u + \frac{1}{2\tau}\left(C_{D\alpha} - C_L\right)\alpha + \frac{C_L}{2\tau}\theta = 0$$

$$\underline{\text{MOMENT}}: \quad \frac{1}{\tau^2}\left(C_{m_\alpha} + C_{m_{d\alpha}}\right)\alpha + \frac{1}{\tau^2}C_{m_{d\theta}}d\theta - h\,d^2\theta = -\frac{C_{m_\delta}\delta}{\tau^2}$$

where u, α, and θ are increments of velocity, angle of attack and pitch angle, and the operator "d" is d( )/dt, where time, t, is in seconds.

These three equations of motion are the stick-fixed equations, and as such do not take into consideration elevator hinge moment parameters and inertia effects of freeing the elevator. However, as the stick fixed phugoid can be very closely approximated by the use of a positive control stop in the aircraft, and as the aircraft was small with light control surfaces it is believed that stick - free effects are thereby minimized in normal flight maneuvers.

These equations of motion are in the three unknowns, u, α, and θ, the first of which is the non-dimensional output variable for the problems of this report. The quantity, $\frac{C_{m_\delta}\delta}{\tau^2}$, is the forcing function or input

variable for the problem.

If these simplified equations of motion accurately represent the motion of an aircraft, then an elevator forcing function of a particular form should cause u, α, and θ to vary in the same manner in both the actual aircraft and in the equations. This is, of course, assuming that the aerodynamic and stability parameters used in the equations are correct. Also the converse should be true. If one variable, say u, were to be manipulated in a particular manner in the air, and the forcing function measured, then a similar manipulation of u in the equations of motion should reveal an identical forcing function.

PROCEDURE AND DISCUSSION

The aircraft center of gravity and moment of inertia were obtained by weighing the aircraft and oscillating it about its jack points. This procedure was carried out with the aircraft in its flight configuration - pilots positioned, fuel aboard, instrumentation installed, and gear and flaps up. A schematic diagram, together with the calculations performed, may be found in the Sample Calculations section of the report.

Stability and aerodynamic parameters required for the equations of motion were determined by calculation or from Refs. 3 or 4 as indicated in Appendix B. For the purpose of these calculations, it was assumed that the aircraft was flying at 5000 feet pressure altitude at a true air speed of 110 mph. This was done in order to conform with the flight test conditions in Parts I and II of the report and to allow comparison with the data of those sections. These parameters when substituted into the simplified equations of aircraft motion shown in the theory section of Part III give:

<u>LIFT</u>: $.400\,u + (.213 + d)\alpha - d\theta = 0$

<u>DRAG</u>: $(.038 + d)u - .0420\alpha + .200\theta = 0$

<u>MOMENT</u>: $(.275 + .450d)\alpha + .10\,d\theta - .0642d^2\theta = .655\,\delta$

These three equations were set up on an electronic analog computer as indicated in Fig. 18 and solved simultaneously using a step function input voltage corresponding to an incremental step function of .8 degrees elevator angle.

The velocity response to this step function input was recorded on a Brush Pen Recorder. It was desired to record the actual velocity change, V, instead of the non-dimensional incremental velocity, u, so the relationship $\Delta V = Vu$ was used to calibrate the velocity trace. As a step elevator input of $.8^{\circ}$ was used as a forcing function the velocity response of the computer system could be compared with the actual response of the aircraft during Run 2172 as shown in Fig. 3 and discussed in Part I. The actual aircraft had a period of 36 seconds and a time to damp to half amplitude of 25 seconds during this run while the solution of the simplified equations of motion by the computer indicated a period of 31 seconds and a time to one half amplitude of 58 seconds. These discrepancies may be attributed to the following reasons: the aircraft is actually non-linear in the speed range considered, that stick fixed conditions did not actually exist due to control cable stretch or flexibility, and that propeller effects were not included in the computations for the C_D used in the simplified equations. Ref. 5 indicates the necessity of including this last factor.

In an effort to make the computer velocity response to a step elevator input more nearly correspond with that of the aircraft the value of $\frac{C_D}{\tau}$ was changed to .1 (an unreasonable value). This gave a period of 31 seconds and a time to damp of 29 seconds which was a close enough approximation for the purposes of this section. The expression for the aircraft transfer function as obtained in Part I of the report was not used, as it was desired to check the classical equations as indicated above.

A mock-up to simulate the elevator control of the test aircraft was constructed and is illustrated in Figs. 15 and 16. The mock-up consisted of a wooden frame supporting a Navion control column. The fore and aft movement of the control column was restrained by 4 elastic strips which roughly simulated the stick force required to move the elevator in flight. The control column was connected through a pulley system to a precision potentiometer which enabled picking off positive or negative voltages corresponding to fore and aft control (or down and up elevator) movement, respectively. The ratio of the control displacement to elevator angle change in the mock-up was adjusted (as explained later) to approximately the same value as in the actual aircraft. A meter whose deflection was proportional to u (and thus to ΔV) was displayed in front of the pilot immediately above the control column on the mock-up. One side of this meter

was connected to ground on the computer and the other side was connected to the u output of the computer. This meter is shown in Figs. 15 and 16 and in the schematic Fig. 17. It should be noted that the scale of the meter read from -100 to +100 with 0 in the center position. Thus, deflections moving toward the right indicated increases in air speed while those moving to the left indicate decreases in air speed. When the stick was in its neutral position there was no δ input and consequently no u output from the computer, and the u meter read zero.

In order to record the time history of the change in velocity and the pilot's movement of the control column, a two channel Brush Pen Recorder was connected to the computer to record the input δ and the output $u \approx \Delta V$. The voltage supply across the potentiometer was adjusted so that a control movement which produced 1° of elevator angle change in the airplane caused approximately 1 volt = 1 degree input to the computer. The controls of the recorder were adjusted to give the scale factors indicated in Figs. 19 and 20. Thus a 60 unit deflection on the u meter corresponded approximately to a 13 mph change in velocity while a 120 unit deflection of the meter was roughly a 26 mph change. It should be noted that these calibrations are close approximations and not exact.

The pilots practiced "flying" the mock-up by starting with the control column in the neutral position (u meter

reading zero) and attempting to decrease the meter reading (and thus the velocity), through movement of the controls, to the -60 position (ΔV = 13 mph) in a smooth and expeditious manner and arriving with no overshoot or undershoot. A recorder trace of a run of this type is shown as Fig. 19. Other runs were performed in which the pilot started from a stabilized value of meter deflection of + 60 and attempted to decrease it to -60. This corresponded to a decrease in air speed of 26 mph. A record of this type air speed change is shown in Fig. 20. In all runs it was found very difficult to move the controls smoothly and evenly due to the large amount of stiction in the mock-up. It was never possible to arrive at the final meter reading without some oscillations.

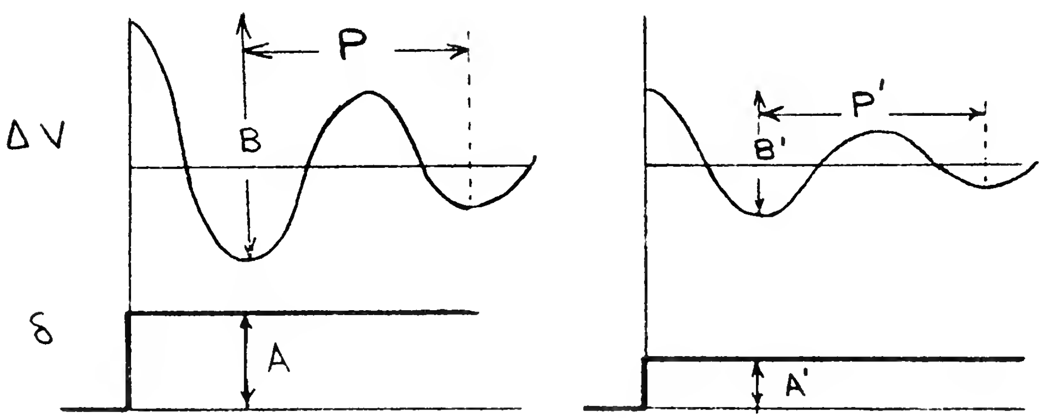
Observation of one pilot "flying" the computer by another, plus discussion between pilots as to how they flew the computer yields the following information. The initial control movement made in both types of runs to initiate the speed change was one which, in the memory or experience of the pilot, corresponded to that which he had made in the test aircraft to produce a similar air speed change. Control movements subsequent to the initial movements varied in each type of run. In the case of the 60 unit change of the u meter the initial stick

position was held until about 50 units of change had occurred, at this time about half the control displacement was removed in order to avoid overshoot. This was almost immediately followed by a backward movement of the control column to avoid undershoot. Then followed a random motion of the control in an attempt to quiet the oscillations of u about the final value. In the case of the 120 unit change the initial control movement was held until the needle slowed perceptibly (which occured short of the desired change), at this time an additional rearward movement of the controls was made to further increase the value of u. This was followed by random motion of the elevator to stop the oscillations about the final value.

An examination of the records of these two type runs, Figs. 19 and 20, immediately reveals the following things. The pilot handled the elevator control in the mock-up far more roughly than he did in the actual airplane. This can probably be explained by the lack of "G" resulting from the control movement, the failure to properly simulate the control forces, and the large amount of static friction in the system. The pilot reached his final meter reading within 13 to 15 seconds after his initial control movement regardles of whether he was changing air speed 13 or 26 miles per hour. The pitch rate of the aircraft would be higher in making large

speed changes than in the smaller change. The pilot experienced difficulty in settling down to his new meter reading without oscillations.

Before attempting to explain these phenomena it is necessary to consider the velocity response to an elevator step function in both the test aircraft and the computer equations in more detail. As the equations of motion in the computer are linear, an elevator step function input will cause the phugoid mode to be excited and the amplitude of the oscillations will be in a definite ratio to the size of the step input. If a bigger step is applied, a bigger oscillation will result, but this ratio will remain constant, and, in addition, the period and the time to damp to half amplitude will remain the same. This is illustrated below.



$$\frac{B}{A} = \frac{B'}{A'} \quad \& \quad P = P'$$

The aircraft is not linear over the range of speeds tested in Part I, however, the same relationship is approximately true as is shown in Table I and Figs. 2 and 3.

As mentioned before, the aircraft equations of motion used in the computer did not exactly match the actual aircraft transfer function found in Part I. Consequently, Figs. 2 and 3 can be used only for qualitative comparison with the mock-up-computer system results. However, it can be seen that in "flying" the computer, the pilot put on an initial step function and waited to see how it affected the velocity response. The effect he witnessed was essentially the first oscillation of the phugoid mode. In the case of the 13 mph decrease in air speed (Fig. 19), he generally put on an initial step elevator deflection that caused a phugoid oscillation of too great an amplitude which would have carried him beyond his desired speed had he not applied opposite control as he realized he was approaching his final speed too rapidly. The opposite control, which was applied at about 12 or 13 seconds, together with the decrease in the rate of speed change which occurred as the phugoid oscillation approached its maximum (at about 15 or 16 seconds) enabled him to reach an airspeed close to his desired one. However, at this time the phugoid oscillation caused the air speed to commence increasing. This

necessitated a large amount of up elevator (back control) to stop. Thereafter, the pilot's control movements were "desperation" movements to stop the succeeding small oscillations. In the case of the large air speed change (Fig. 20), the pilot usually applied an initial step input that was not sufficient to cause a phugoid oscillation which would take him to his desired air speed. Thus, as he watched his air speed needle move, the rate of change was satisfactory for the first 12 or 13 seconds until the rate of change of air speed began to decrease due to the phugoid approaching the maximum of its first oscillation, and it seemed that he would not reach his desired speed. As a result, he added more up elevator which brought him to his final speed and also partially compensated for the change in phugoid oscillation tending to increase his air speed. As this increase in speed due to the phugoid progressed he added more up elevator. By this time he had the mock-up fairly well under control, and his subsequent movements were not as radical as in the smaller air speed change.

From the preceeding discussion it may be seen that the primary reason the pilot reached his new air speed in 13 to 15 seconds was due to his step function elevator input. This input initiated a phugoid mode whose first oscillation took 15 to 16 seconds to reach a maximum, and whose amplitude was approximately that of the desired air speed change. The oscillations of the air speed about

the final desired value, and the associated radical control movements occurred largely as a result of the reversal of the phugoid oscillation which tended to cause a reversal in the direction of air speed change.

The increased pitch rate in the case of the larger air speed change may be considered to stem from two causes. First, the pilot when flying the computer had no sensation of "G" force and consequently had no hesitation in applying an elevator step change of considerable magnitude immediately, rather than applying it gradually as he did in the aircraft. Secondly, the pilot was reluctant to put on as much control deflection in the case of the small speed change as he would use in a larger change for fear he would overshoot.

An attempt was made to correlate the time history of velocity and elevator response as recorded from the mock-up runs, Figs. 19 and 20, with those recorded during the smooth air speed changes in the test aircraft in flight (Figs. 5 through 11). Because of the difference in the values of air speed changes and elevator inputs on the mock-up and in actual flight this was difficult to do. However, it was noted that the velocity response curves for runs 2167 and 2170 were almost identical in character with the 26 mph mock-up changes during the first 9 seconds. After this time, the flight test velocity response commenced to level out as the pilot approached a final air

speed 5 mph less than that made on the mock-up. This initial coincidence does show the similarity of the initial transient behavior of the phugoid in each case. No comparison between the elevator movement could be made as the movement of the elevator controls in the mock-up was far more erratic and rough than when in the air. Had the mock-up been more carefully constructed so that the control forces were more nearly correct, and had the static friction been removed from the system, smoother elevator control would have been realized.

When flying the aircraft and making smooth air speed changes, the pilot did his best to concentrate on the air speed indicator in an attempt to have this as his only source of information when making the air speed change. He did, in fact, have other means at his disposal to help evaluate the manner in which he was performing the maneuver. The "seat of his pants" gave him information as to whether or not the "G"load on the aircraft was excessive or uncomfortable and his peripheral vision informed him of the aircraft attitude and pitch rate. With these added sources of information at his disposal he was able to overcome changes in sign of the phugoid oscillation with no conscious effort on his part by merely exerting pressures on the controls rather than making actual control displacements. It is felt that if the pilot had had additional information concerning pitch rate

at his disposal while making the mock-up air speed changes his control positioning would have been less erratic.

Another, and still better, method of making a more reasonable correlation between the flight test results and the computer mock-up runs is the following. Conduct the flight tests with the pilot essentially flying blind, and with all instruments covered except the air speed indicator. This would remove all information sources except "G" and that portrayed by the air speed indicator. Instead of the classical equations of motion being used in the computer, they should be replaced with the actual aircraft transfer function as determined in Part I. If these techniques were followed and a mock-up of better design were incorporated, it is felt that a reasonable and interesting comparison might be made.

RESULTS

The analog computer solution of the classical equations of stick fixed motion were found not to accurately represent the aircraft velocity response when the speed change is of the order of 10 to 25 mph. This may be attributed to the following: the aircraft is not linear in the speed range considered; that stick fixed conditions did not actually exist due to control cable stretch or control flexibility; that propeller effects were not considered in the computation of the drag coefficient. The phugoid period as computed by the classical equations was within 15% of the value as determined by flight test. The time to damp to half amplitude computed by the classical method was greater than that found in flight test by a factor of 2. These values of period and time to damp could be changed to more nearly agree with the actual aircraft by changing the value of drag coefficient by a factor of 2 to 3. In future work of this sort it is felt that the value of the aircraft transfer function as determined by flight test in Part I would be more satisfactory than trying to adapt the classical equations of motion.

It was found that insufficient attention had been devoted to the mock-up construction. The static friction in the control column was too high. This could be reduced by the use of proper bearing surfaces. The control forces were not similar to those encountered in the actual aircraft. Better spring restraints on the control column

and a type of "Q box" would remedy this situation. The above considerations added materially to the difficulty of "flying" the computer. The method of presentation of the air speed data on the mock-up was completely adequate, and similar to that which the pilot encountered in normal flight.

Changing air speed on the mock-up smoothly and with no overshoot or undershoot proved extremely difficult. In addition to the mock-up limitations enumerated before, certain other factors contributed to this difficutly. The pilot positioned his controls initially as a step function elevator input in accordance with his memory as to how he had displaced the controls in the aircraft. Then, in the absence of any information as to "G" or pitch rate, he was forced to adopt a "wait and see" attitude as the phugoid mode progressed. In general, the amplitude of the first oscillation of the phugoid mode which was excited by the initial control deflection was such as to reasonably approximate the final air speed change desired. Subsequent oscillations about the final value of air speed seemed to be closely associated with the phugoid oscillation reversing and causing an increase in air speed. Control movement in this area seemed to be random in nature. Pilot opinion indicated that it was easier to stabilize the mock-up after a large air speed change. This may be due to the fact that the pilot had more experience in making large air speed changes on the computer, or that, as his initial

control input in the case of the large air speed change was not sufficient to cause a phugoid oscillation that would overshoot his final airspeed, he was forced to add additional up elevator to get to his final speed and this additional elevator helped overcome the tendency for the air speed to increase as the phugoid oscillation reversed.

As the motion of the elevator in the computer mock-up runs was so erratic, no attempt was made to compare them with the elevator motion in flight test. It was felt that with an improved mock-up design, and a flight test procedure that limited the sources of pilot information to the air speed indicator and to "G" would allow intelligent correlation of the data.

CONCLUSIONS

The classical equations of stick fixed aircraft motion do not accurately represent the aircraft velocity response when the speed range is of the order of 10 to 25 mph.

It would have been better to have used the aircraft transfer function as obtained in Part I in lieu of attempting to adapt the classical equations.

Static friction in the mock-up control was too high.

Control forces in the mock-up should have more nearly simulated those of the test aircraft.

In attempting to change air speed on the mock-up the pilot applied a step function elevator input which corresponded in his mind to the stick displacement required for a similar air speed change in the test aircraft. This elevator deflection excited the phugoid mode, and the magnitude of the first oscillation brought him very close to his desired air speed change.

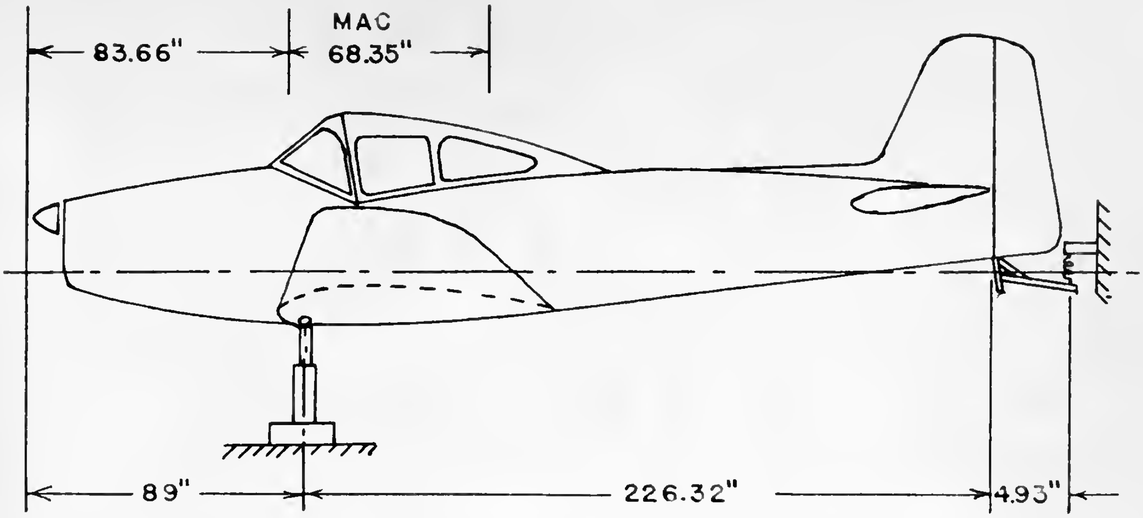
Oscillation about his final air speed with the accompanying erratic control movements represent an effort to nullify the effect of the continuing phugoid mode.

No comparison of the elevator movement could be made between the in-flight air speed change and the mock-up speed changes.

No comparison of the elevator movement could be made between the in-flight air speed change and the mock-up speed changes.

The absence of other information such as acceleration and pitch rate made the pilot's control movements rough and erratic, and caused him to adopt initially a "wait and see" attitude followed by a slam bang "I can stop that" philosophy.

In any future investigations it is recommended that the mock-up construction be improved, and that either more information be displayed on the mock-up or that flight tests be conducted where the pilot is flying blind with all instruments except the air speed indicator covered in order to allow for more intelligent correlation of air speed and elevator data. The aircraft transfer function as determined in Part I should be used in the mock-up computer system in future tests.



DETERMINATION OF AIRCRAFT CENTER OF GRAVITY

AND MOMENT OF INERTIA

Weight on jack points:	2,580 lbs.
Weight on tail skid :	168 lbs.
Total Weight:	2,748 lbs.

$$(2,580)(89) + (168)(315.32) = 2,748X$$

$$X = \frac{229,900 + 52,900}{2,748} = \underline{102.9 \text{ inches}}$$

$$\% \text{ MAC} = \frac{102.9 - 83.66}{68.35} = \underline{28.1\% \text{ MAC}},\ X_{cg} = \underline{.281}$$

Assuming an undamped oscillation,

$$I_{jack} = \frac{P^2 k l^2}{4\pi^2}$$, where "k" is the

spring constant, "l" the spring lever arm and "P" the period of the oscillation.

Test data: $k = 70.65$ lb./in.; $P_{av} = .666$ sec.

$$I_{jack} = \frac{\overline{.666}^2 (70.6 \times 12)(231.25)^2}{(144)(4\pi^2)}$$

$$= 3{,}521.9 \text{ slug ft}^2.$$

Transfer I_{jack} to I_{cg} (I_y):

$I_Y = I_{jack} - mr^2$, where m is the aircraft mass and "r" the distance from the jack points to the CG.

$$I_y = 3{,}521.9 - \frac{2{,}748 \times \overline{13.9}^2}{32.2 \times 144}$$

$$= 3{,}521.9 - 114.7$$

$$\underline{I_y = 3{,}407.2 \text{ slug ft}^2.}$$

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APPENDIX A

INSTRUMENTATION AND CALIBRATION

The test airplane, a standard Ryan Navion, was instrumented to record time histories of elevator angle, stick force, and velocity. These variables were recorded with a Consolidated Engineering Corporation, type 5-116-P3-14 recording oscillograph equipped with galvanometers which deflected one inch per 12.4 micro amperes of current.

The design criteria, operation, and calibration procedures for the elevator angle and stick force circuits are fully outlined in Appendix A of Ref. 3. Wiring diagrams and schematic diagrams for these circuits appear as Figs. 5, 6, and 7 of Ref. 3. Both elevator angle and stick force were calibrated before and after each test flight. The calibration of both circuits was found to be stable throughout the period of flight tests, and the calibration curves for elevator angle and stick force are shown in Figs. 25 and 26, respectively. It should be noted, that the stick force sensitivity was decreased radically during runs 2165 to 2174 as this quantity was no longer needed, hence the calibration does not apply to to these runs.

The design of the velocity measuring and recording system was established by the desire to record velocity on the same medium as the elevator angle and stick force

in order to facilitate reduction of the data. A Staham Differential Pressure Transducer, 0 to .6 psid, Model 827 was borrowed from the U. S. Naval Air Test Center, Patuxent, Maryland. The transducer was mounted on a wing rib under the wing tip fairing of the right wing of the test aircraft. This location enabled the transducer to be connected to the pitot static test boom with a very short length of rubber tubing (one foot), thus minimizing any error due to system lag. All other connections to the test boom were sealed off at the boom.

The transducer was connected to a Consolidated Engineering Corporation Carrier Amplifier, Model 1-118, mounted in the cabin of the test aircraft, through a shielded five-wire lead in the wing. The carrier amplifier fulfilled four functions: (1) supplied voltage to the bridge circuit of the transducer; (2) enabled resistive and reactive balance of the entire system; (3) provided for attenuation and amplification control of the signal output; (4) delivered the signal output to the recording oscillograph. As the only galvanometers available for the oscillograph were too sensitive for proper operation with the carrier amplifier, a 22K resistor was placed in series in the signal output line to the oscillograph to reduce the signal strength, and a 330 ohm damping resistance was placed across the line in order to provide optimum galvanometer response. A schematic of the system is shown in Fig. 21.

As it was desired to record the air speed with the maximum accuracy, and as speeds only in the range of 80 to 140 miles per hour were of interest in the tests, the following method of balancing the system was adopted. The system was balanced both resistively and reactively through the controls on the carrier amplifier at zero miles per hour (i.e. no pressure differential on the transducer, and therefore no signal output from the carrier amplifier to the galvanometers) and with the trace deflection on the oscillograph set at 2.22 inches to the right of center. A calibrated and controllable pressure differential was applied to the transducer through a Meriam Instrument Corporation, Tester Air Speed Indicator, Model A-841. With a pressure differential corresponding to 80 miles per hour on the transducer, the resistive balance was varied so as to bring the trace deflection on the oscillograph back to where zero miles per hour had been, 2.22 inches to the right of center. This in no way effected the linearity of the measurement. Suitable adjustment of the attenuation and amplification controls on the carrier amplifier enabled the oscillograph trace to be positioned approximately 2 inches to the left of center when a pressure differential corresponding to 140 miles per hour was applied to the transducer. A calibration chart of air speed versus trace position was constructed by varying the pressure differential across the transducer in discreet steps.

This calibration procedure was carried out before and after each series of flight tests. Calibration curves for air speed may be found as Figs. 23 and 24. It should be pointed out that these air speeds were calibrated air speeds and had to be corrected for non-standard atmospheric conditions in order to obtain true air speed.

Throughout the series of tests the pilot's air speed reference was a sensitive type air speed indicator which received its inputs of static and total pressure from a different source than that which supplied the transducer. As air speeds measured from these two sources agreed within one mile per hour, and as only the absolute value of the air speed change with time was of interest, the arrangement proved to be entirely satisfactory.

APPENDIX B

DETERMINATION OF STABILITY PARAMETERS

All values are determined for a speed of 110 mph true at 5,000 feet altitude .

$$\tau = \frac{m}{\rho Sc} = \frac{83.9}{(002049)(184.2)(161.2)} = 1.375 \text{ sec.}$$

$\tau^2 = 1.892$

$$C_L = \frac{2W}{\rho V^2 S} = \frac{(2)(2,748)}{(.002049)(184.2)(26,100)} = .55$$

$C_L/\tau = .400$

$C_D = C_{D_o} + \dfrac{C_L^2}{\pi Ae}$. From aircraft polar obtained by the Princeton University Aeronautical Engineering Department:

$$C_{D_o} = .026 \; ; \; e = .78$$

Therefore, $C_D = .026 + (.0675)(.303) = .0465$

$C_D/\tau = .0338.$ C_D/τ used was 0.10.

$C_{D_\alpha} = \dfrac{dC_D}{d\alpha} \times C_{L_\alpha}$, C_{L_α} determined from Ref. 4.

$C_{L_\alpha} = .102/\text{deg.} = 5.85/\text{rad.}$

$C_{D_\alpha} = \dfrac{2C_L}{\pi Ae} \times C_{L_\alpha} = .135 \times 5.85 \times .55 = .435/\text{rad}$

$C_{D_\alpha}/2\tau = .158$

$$\mu = \frac{m}{\rho Sc} = \frac{83.9}{(.002049)(184.2)(5.7)} = 38.95$$

$C_{m_\alpha} = C_{L_\alpha}(X_{cg} - N_o) = 5.85(.281 - .370) = -.520$

N_o obtained from Ref. 4.

$C_{m_\alpha}/\tau^2 = -.275$

$$h = \frac{2I_Y}{mc^2\mu} = \frac{(2)(3,407.2)}{(83.9)(5.7)^2(38.95)} = 0.0642$$

$C_{m_{it}} = -\eta_t a_t \bar{V}_t = -(1)(3.72)(.616) = -2.29$

η_t assumed equal to unity.

$$C_{m_{d\theta}} = 1.1\, C_{m_{it}} \frac{l_t}{\mu c} = \frac{(1.1)(-2.29)(15.04)}{(5.7)(38.95)} = -.1705$$

$C_{m_{d\theta}}/\tau^2 = -.901$; (.10 actually used on the computer)

$$C_{m_{d\alpha}} = C_{m_{it}} \frac{l_t}{\mu c}\left(\frac{d\epsilon}{d\alpha}\right)$$. Assuming $d\epsilon/d\alpha = 0.5$

$$C_{m_{d\alpha}} = -2.29\, \frac{(15.04)(.5)}{(38.95)(5.7)} = -.0775$$

$C_{m_{d\alpha}}/\tau^2 = -.410$ (.450 actually used)

Elevator Power:

$$C_{m_\delta} = -a_t \bar{V} \eta_t \tau_e \;;\; \tau_e = \frac{d\alpha_t}{d\delta_e} = .54$$

$C_{m_\delta} = -(.065)(.616)(1)(.54) = -.0216/\text{deg} = -1.24/\text{rad.}$

$C_{m_\delta}/\tau^2 = -.655$

TABLE I

SUMMARY OF TESTS TO DETERMINE AIRCRAFT'S TRANSFER FUNCTION

Run	V_i	V_f	ε	[illegible]	δ	δ/μ	P	$T_{1/2}$
	mph	mph	mph		deg.		sec.	sec.
2136	131.2	108.0	23.2	.177	.82	4.65	36	24
2137	133.0	115.8	17.2	.130	.61	4.72	36.5	25
2150	132.3	110.0	22.3	.167	.78	4.67	36	26
2151	130.0	108.0	22.0	.169	.78	4.62	35	24
2151	131.7	109.2	22.5	.171	.81	4.75	35	24
2171	134.4	117.5	16.9	.126	.59	4.70	37	25
2172	132.0	109.5	22.5	.171	.80	4.70	36	25
2173	132.8	112.0	20.8	.156	.73	4.68	35	25

TABLE II

KEY TO POTENTIOMETER SETTINGS, FIGURES 5, 12 and 18

Figure 5

Pot.	Full Range	Value	Pot. Setting
3	0.1	.01	.10
4	10	1.164	.1164
5	1	.355	.355
6	1	.355	.355
7	1	.420	.420
8	0.1	.0644	.644
13	2	1.0	.50
15	50	31.5	.630

Figure 12

3	1	.522	.522
4	10	1.164	.1164
5	1	.436	.436
6	1	.355	.355
7	1	.703	.703
8	.1	.0644	.644
13	1	.1602	.1602

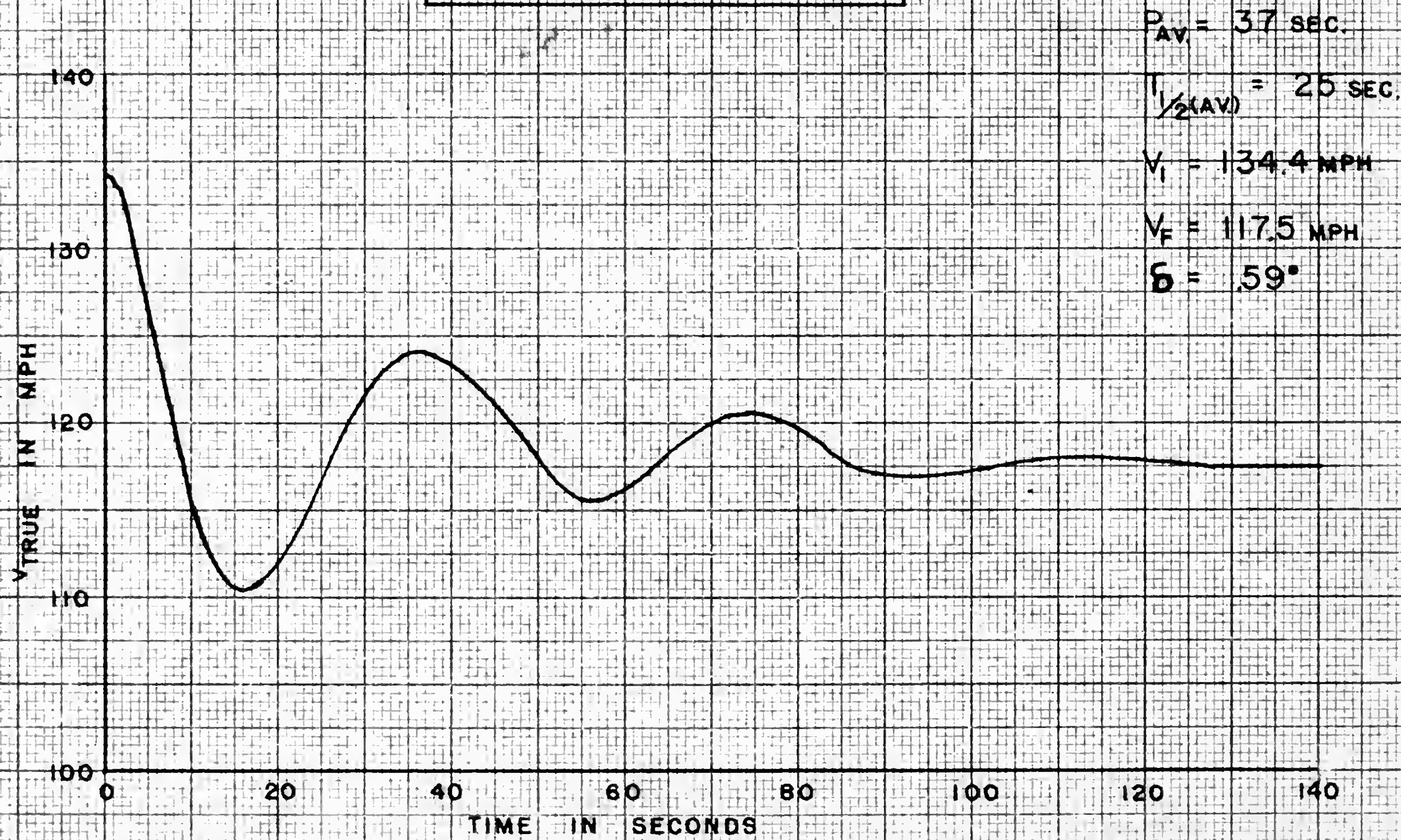
Figure 18

Pot.	Coefficient	Full Range	Value	Pot. Setting
3	C_D/τ	0.1	.10	1.0
4	$C_{L\alpha}/2\tau$	10	2.13	.213
5	$C_{m\alpha}/\tau^2$	1	.275	.275
6	$(C_{D\alpha} - C_L)/2\tau$	0.1	.042	.420
7	C_L/τ	1	.400	.400
8	h	0.1	.0642	.642
9	$C_{m\dot{\theta}}/\tau^2$	1	.10	.10
10	$C_{m\dot{\alpha}}/\tau^2$	0.1	.450	.450
11	$C_I/2\tau$	1	.200	.200
13	$C_{m\delta}/\tau^2$	1	.655	.655

LIST OF FIGURES

FIGURE 1 - TEST AIRCRAFT

FIGURE 2
AIRCRAFT VELOCITY
RESPONSE
RUN 2171



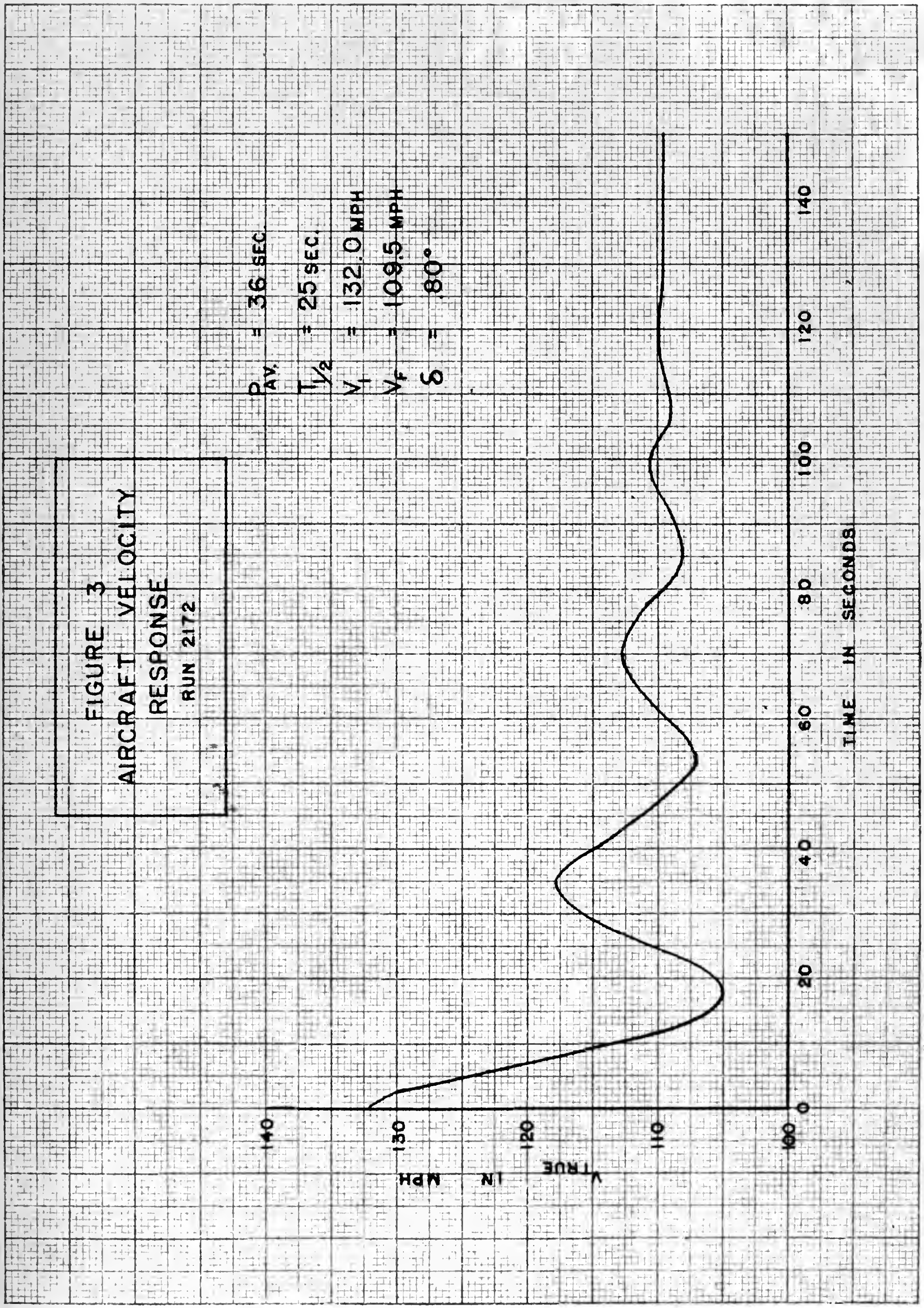


FIGURE 3
AIRCRAFT VELOCITY RESPONSE
RUN 2172

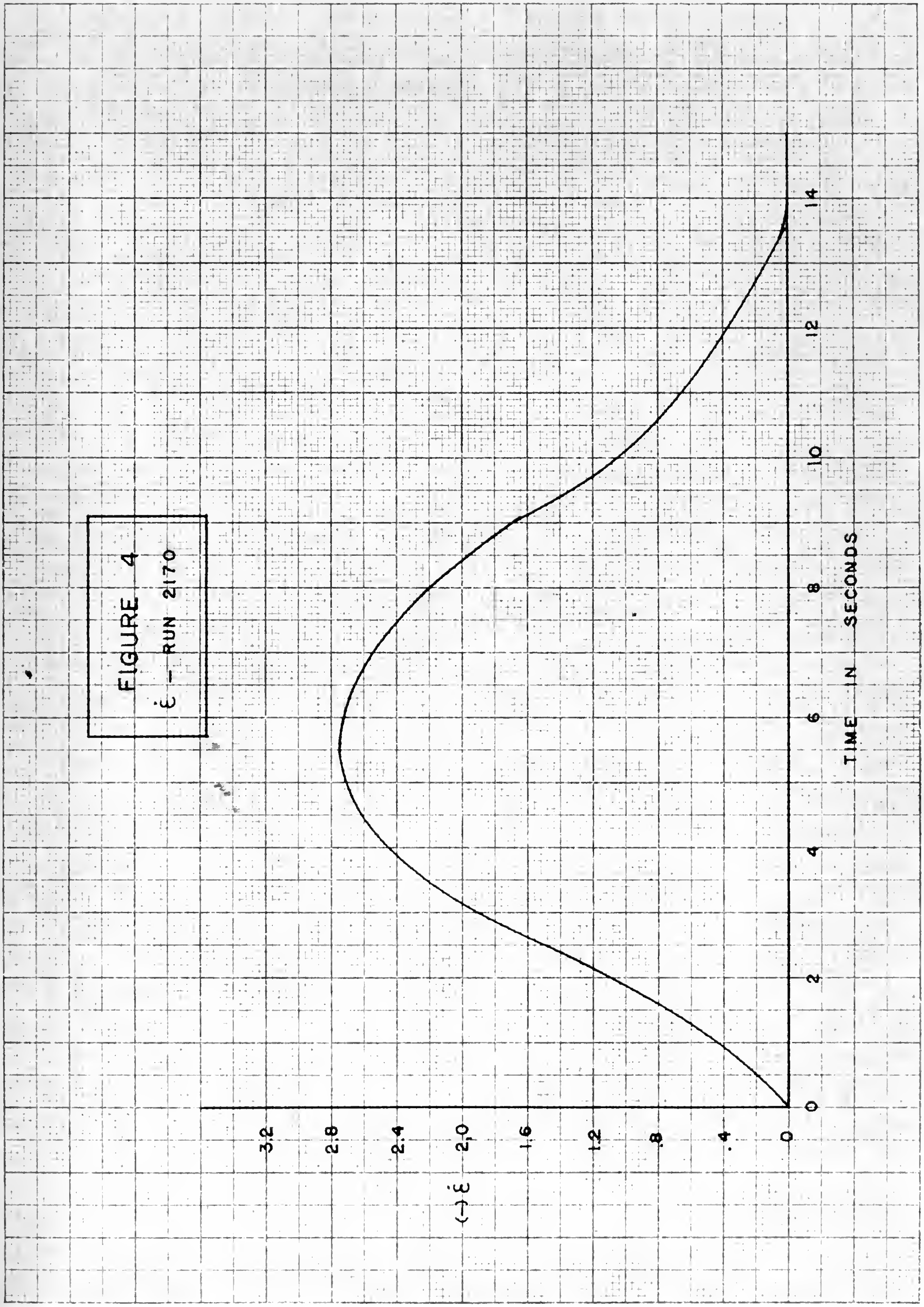


FIGURE 4

Ė — RUN 2170

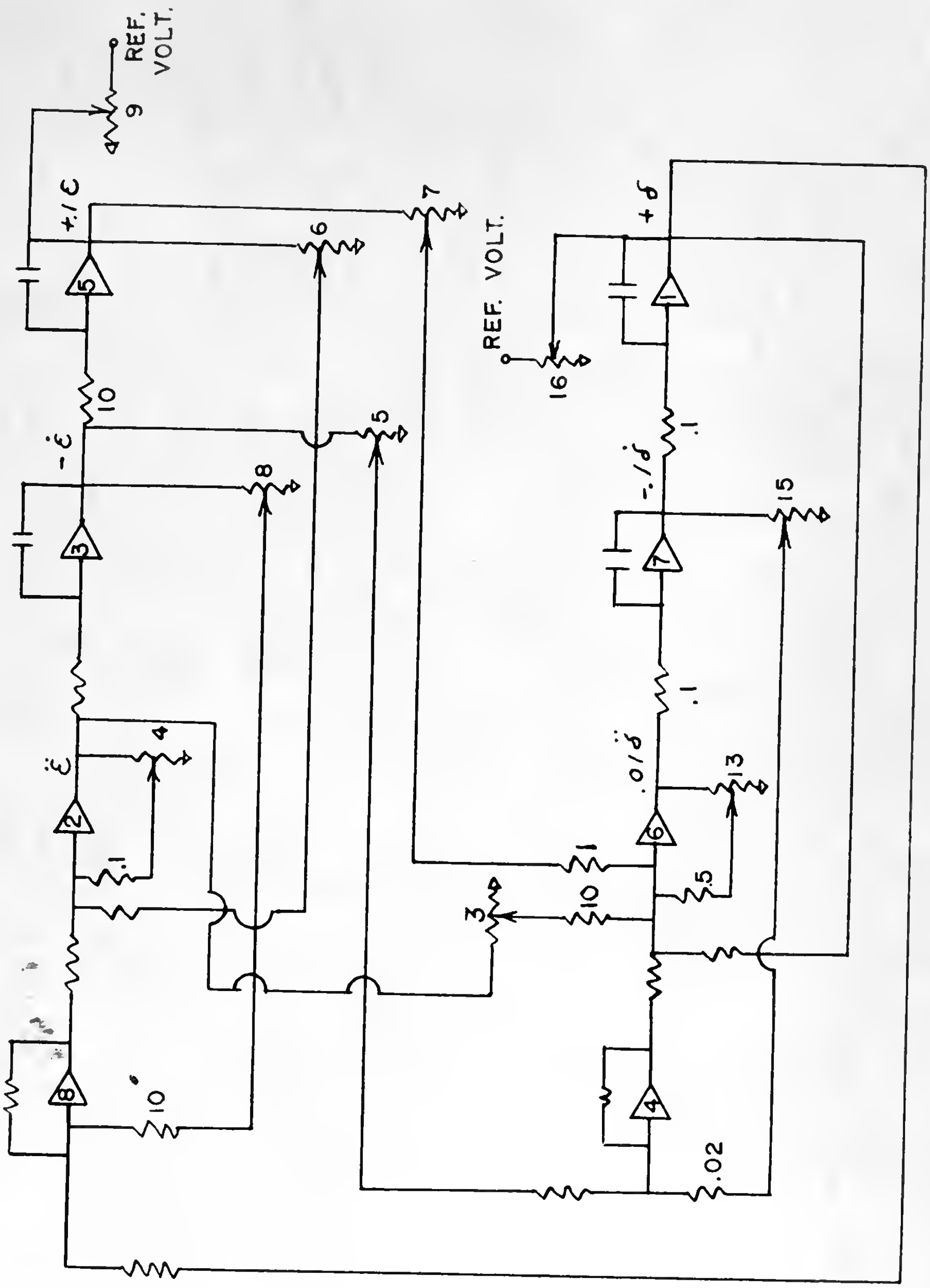


FIGURE 5

ANALOG SCHEMATIC FOR

SECOND ORDER SYSTEM

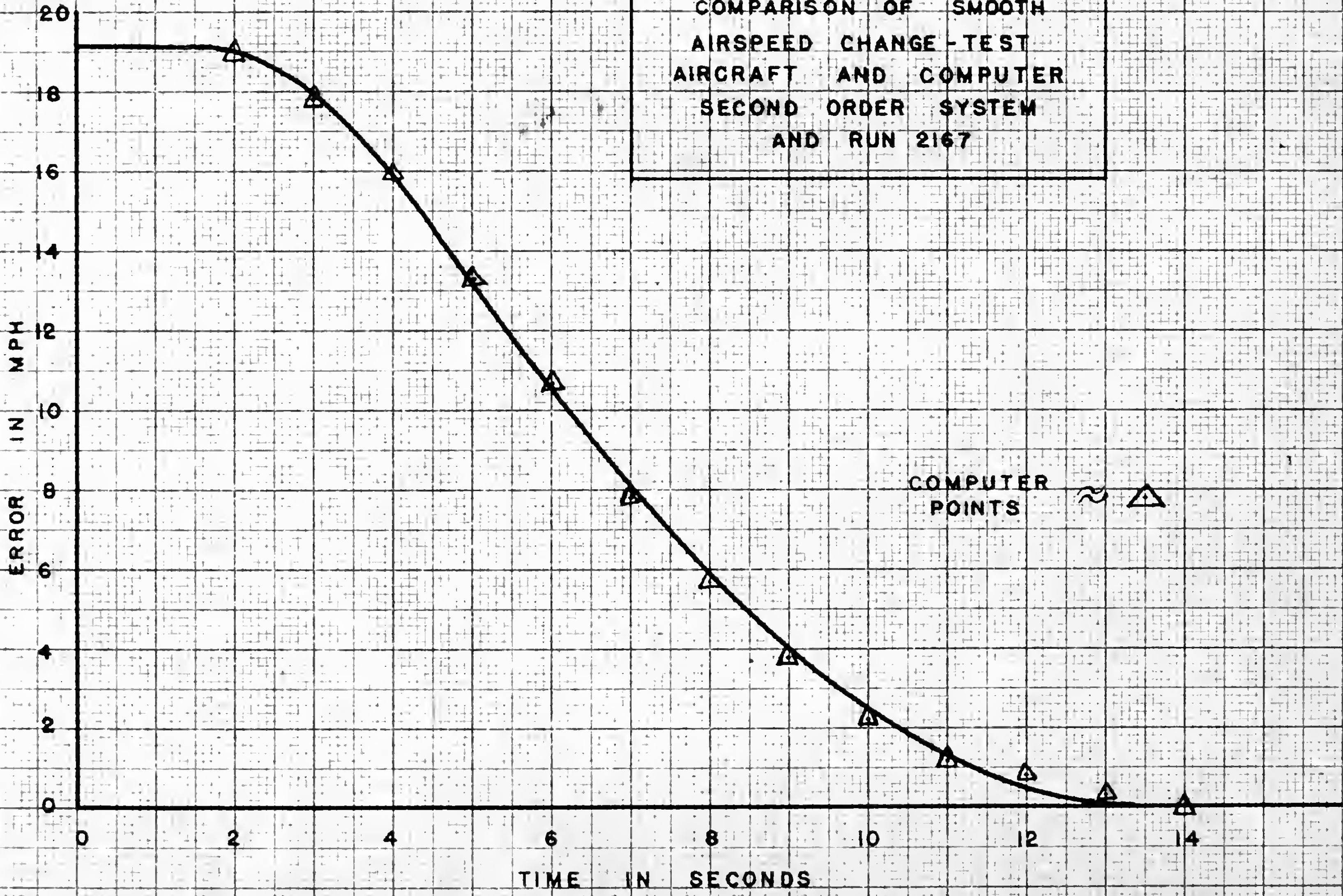


FIGURE 6

COMPARISON OF SMOOTH AIRSPEED CHANGE - TEST AIRCRAFT AND COMPUTER SECOND ORDER SYSTEM AND RUN 2167

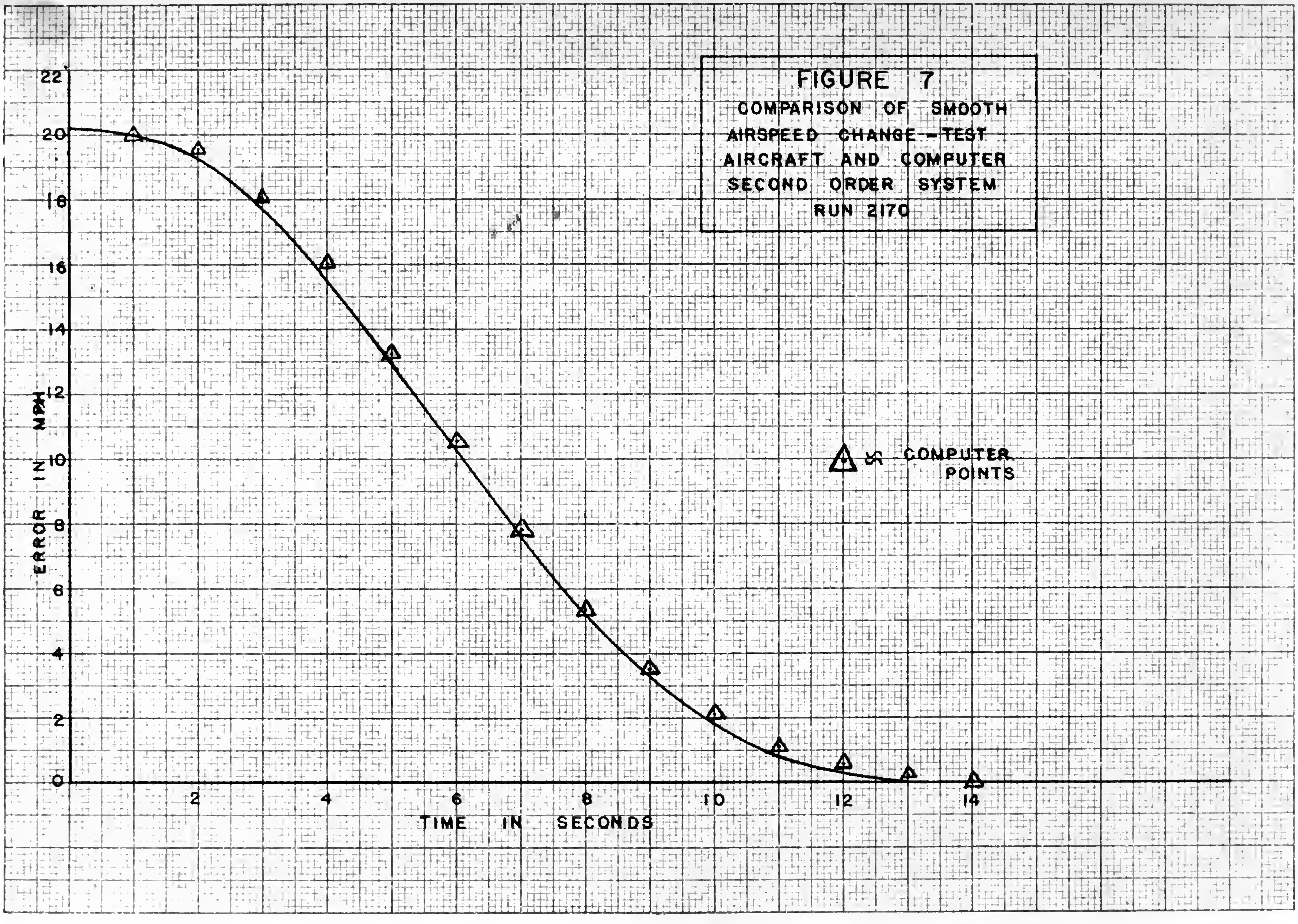


FIGURE 7
COMPARISON OF SMOOTH AIRSPEED CHANGE - TEST AIRCRAFT AND COMPUTER SECOND ORDER SYSTEM
RUN 2170

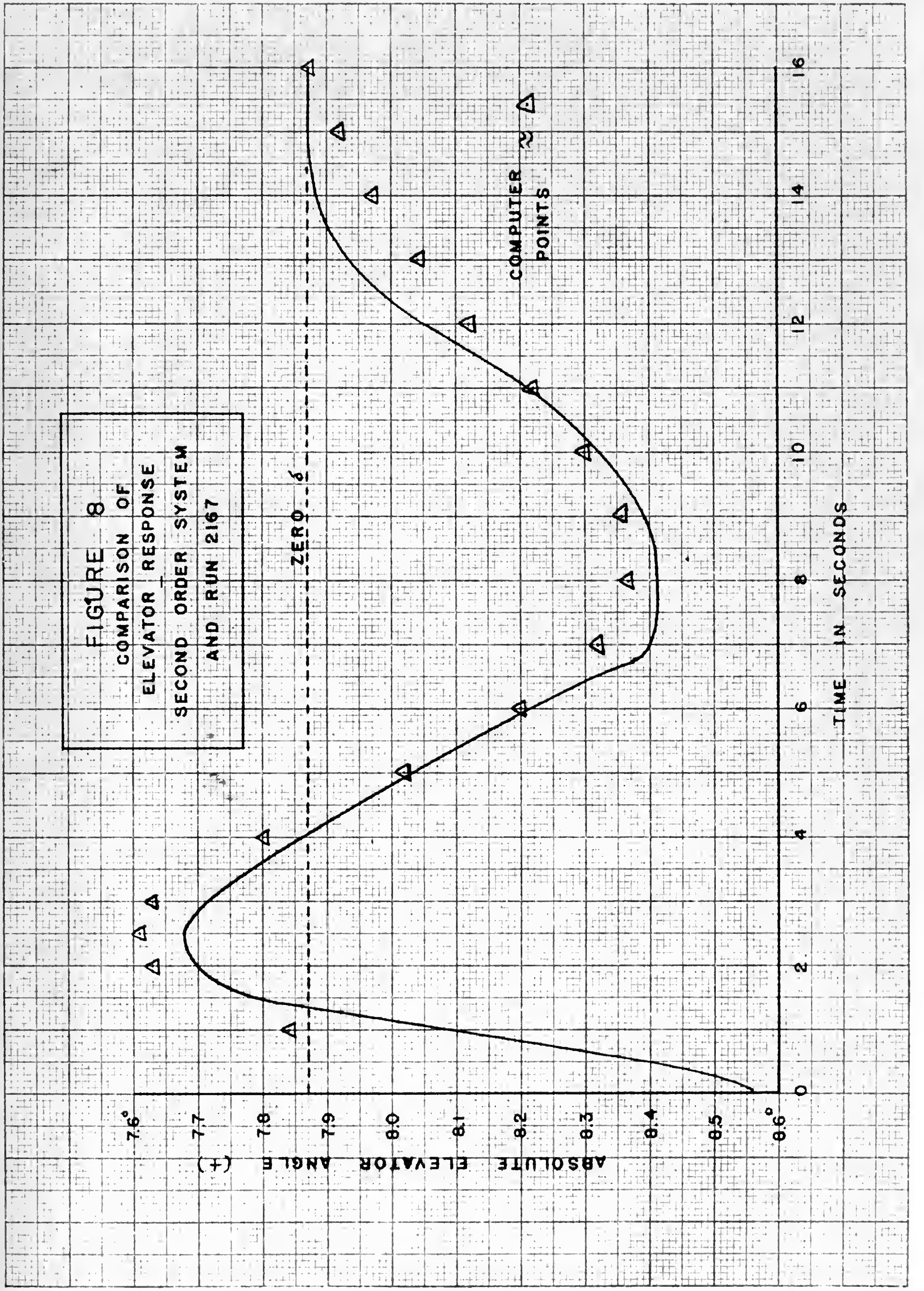


FIGURE 8
COMPARISON OF
ELEVATOR RESPONSE
SECOND ORDER SYSTEM
AND RUN 2167

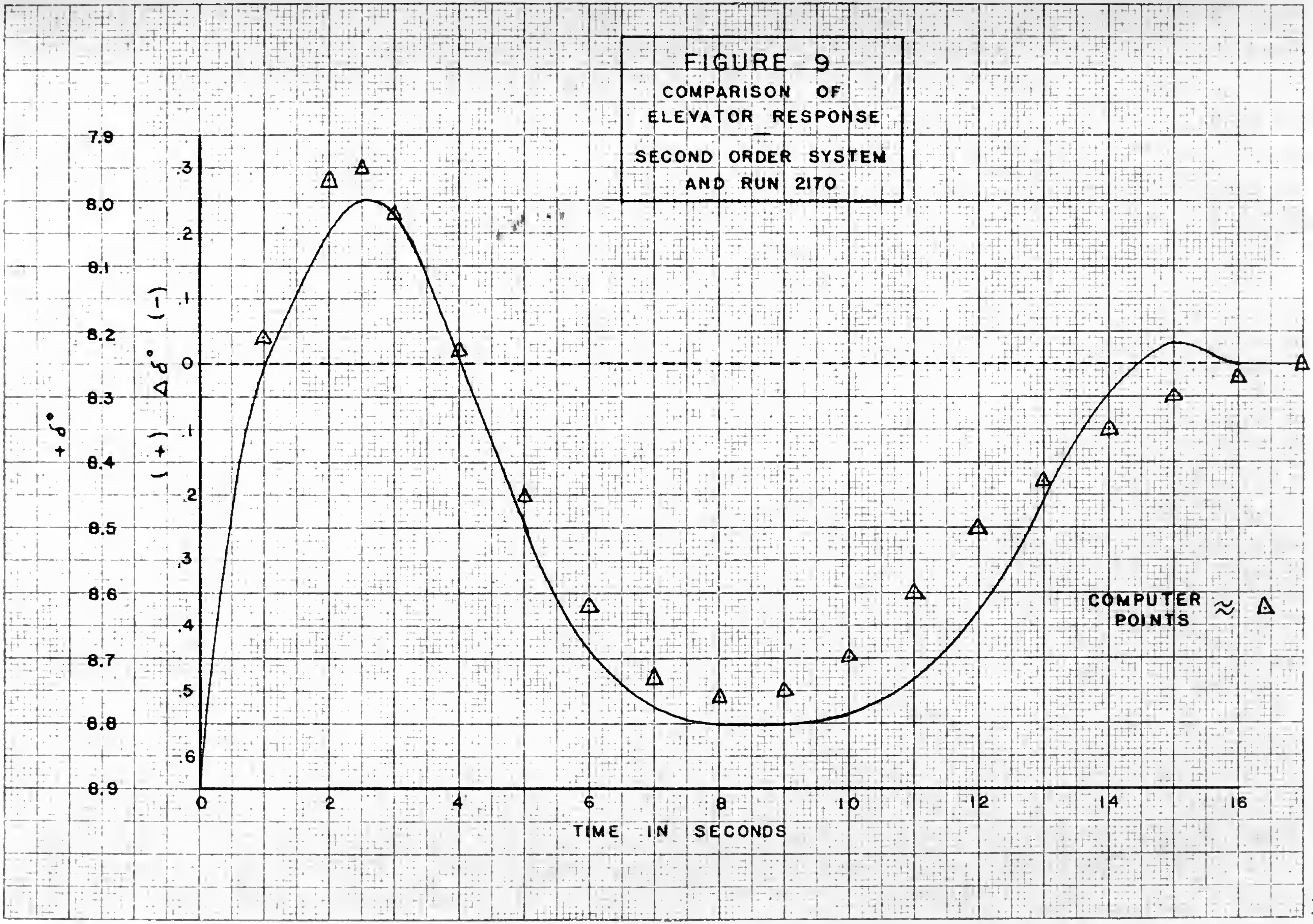


FIGURE 9
COMPARISON OF
ELEVATOR RESPONSE
SECOND ORDER SYSTEM
AND RUN 2170
7.9
8.0
8.1
8.2
8.3
8.4
8.5
8.6
8.7
8.8
8.9
+ δ°
(+) Δδ° (−)
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
0
2
4
6
8
10
12
14
16
TIME IN SECONDS
COMPUTER POINTS ≈ Δ

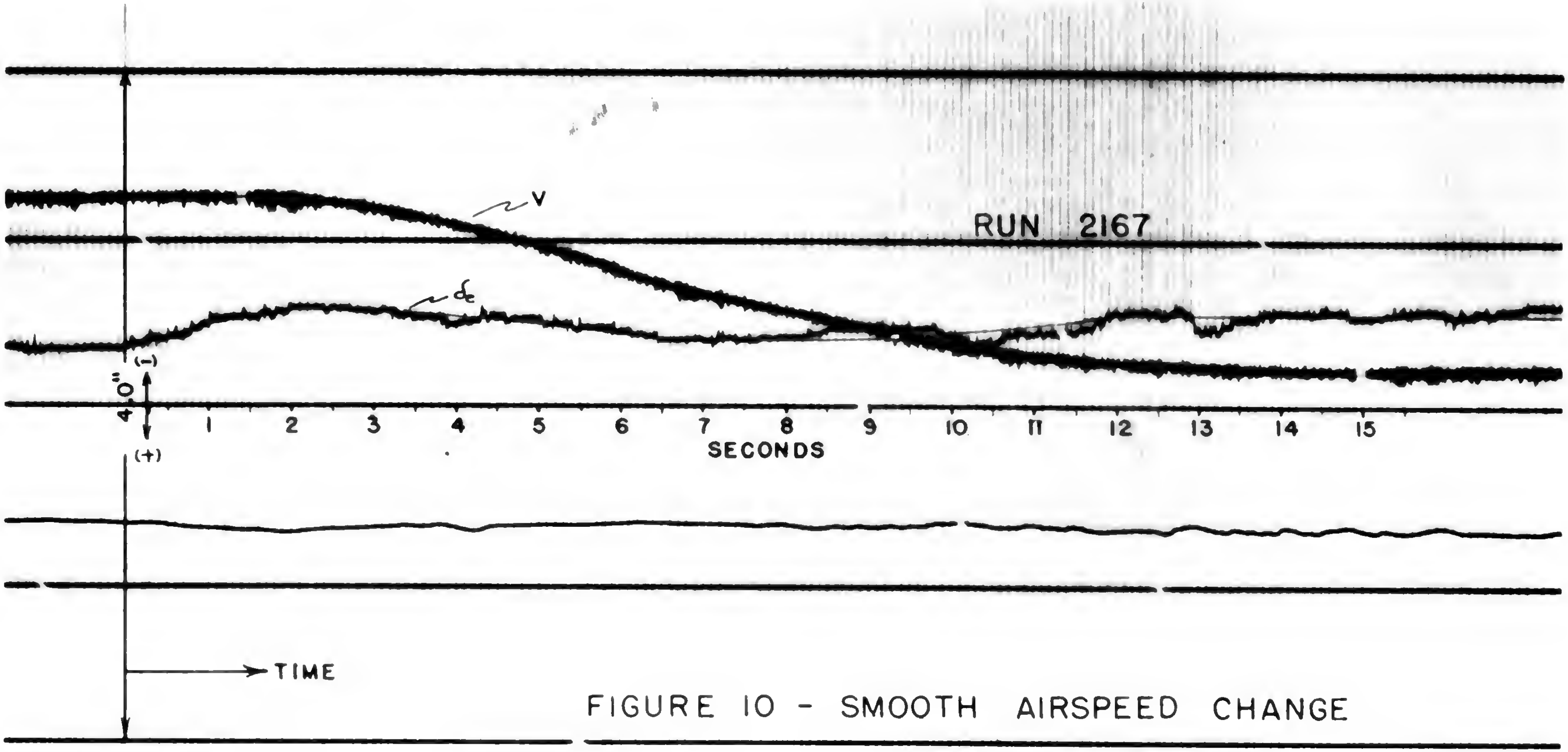


FIGURE 10 - SMOOTH AIRSPEED CHANGE

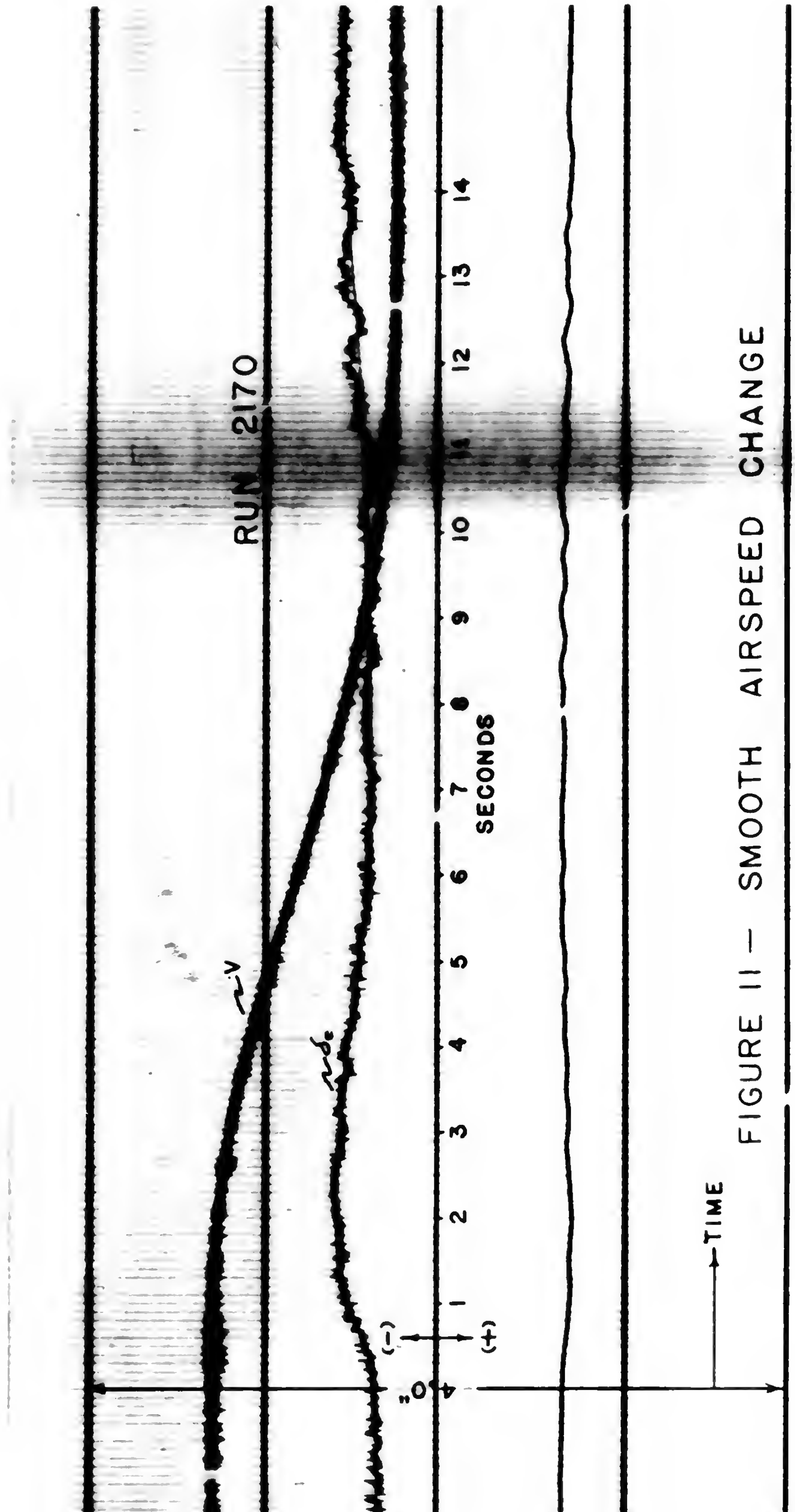


FIGURE 11 — SMOOTH AIRSPEED CHANGE

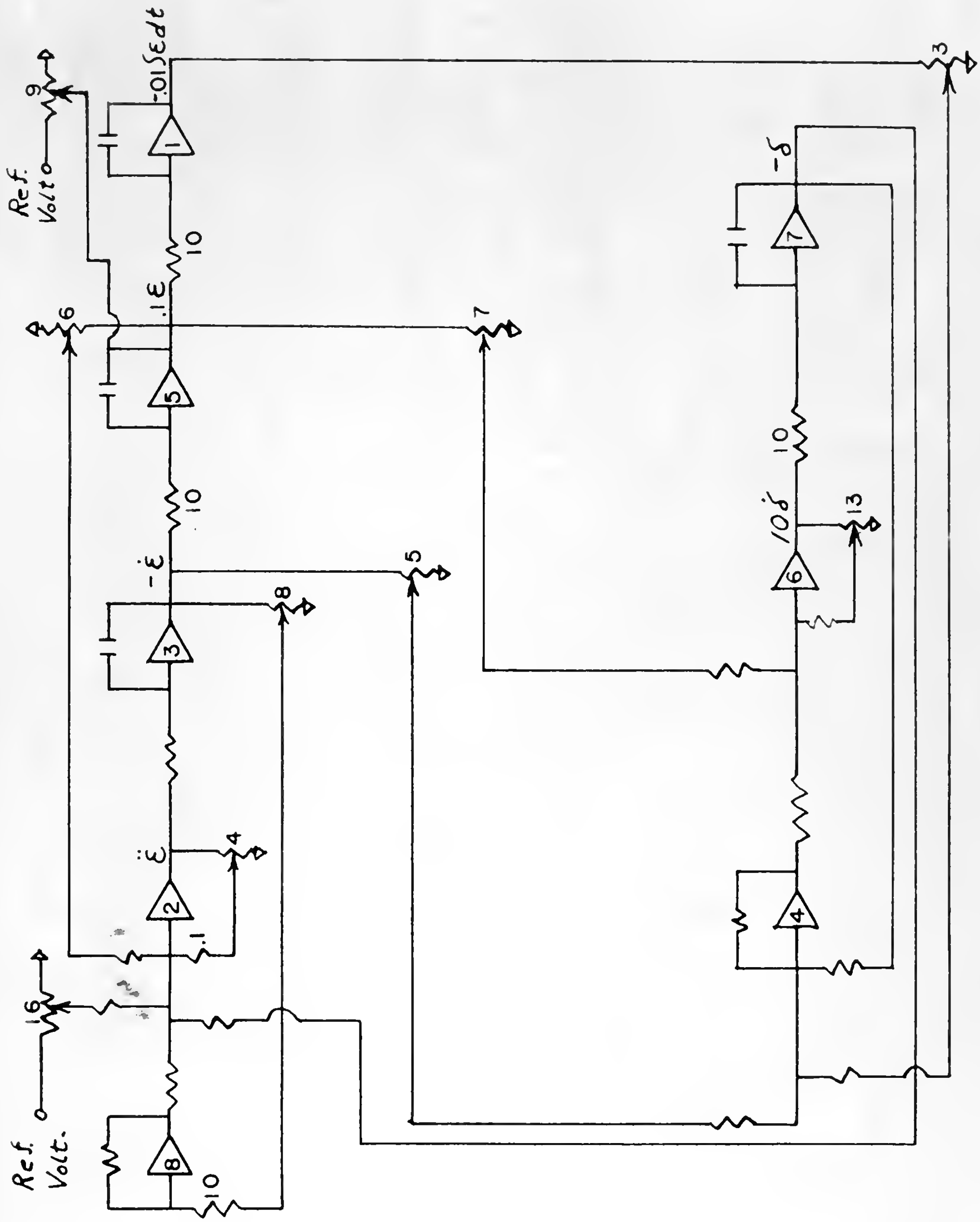


FIGURE 12

ANALOG SCHEMATIC FOR

INTEGRAL SYSTEM

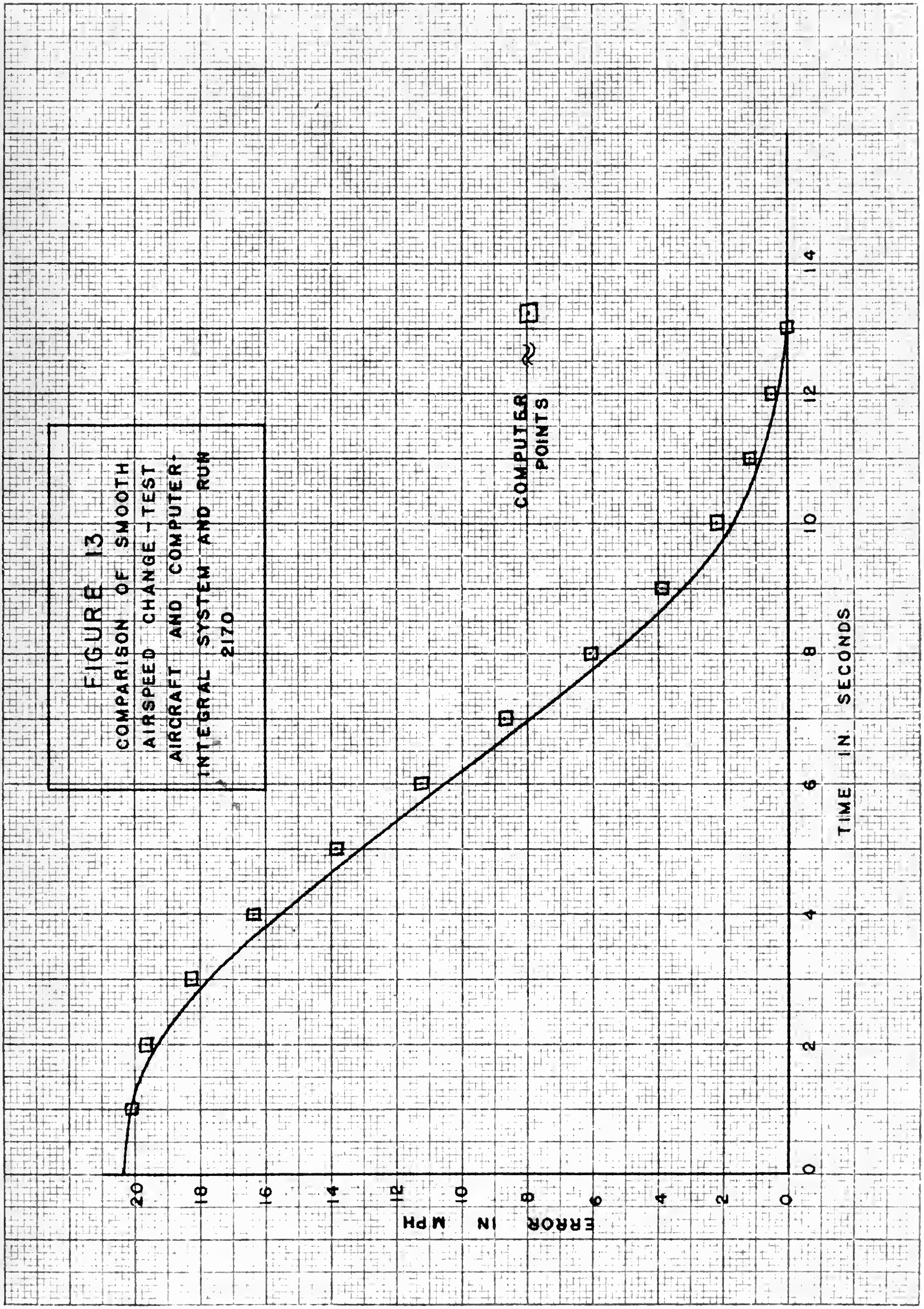


FIGURE 13
COMPARISON OF SMOOTH AIRSPEED CHANGE - TEST AIRCRAFT AND COMPUTER - INTEGRAL SYSTEM AND COMPUTER RUN 2170

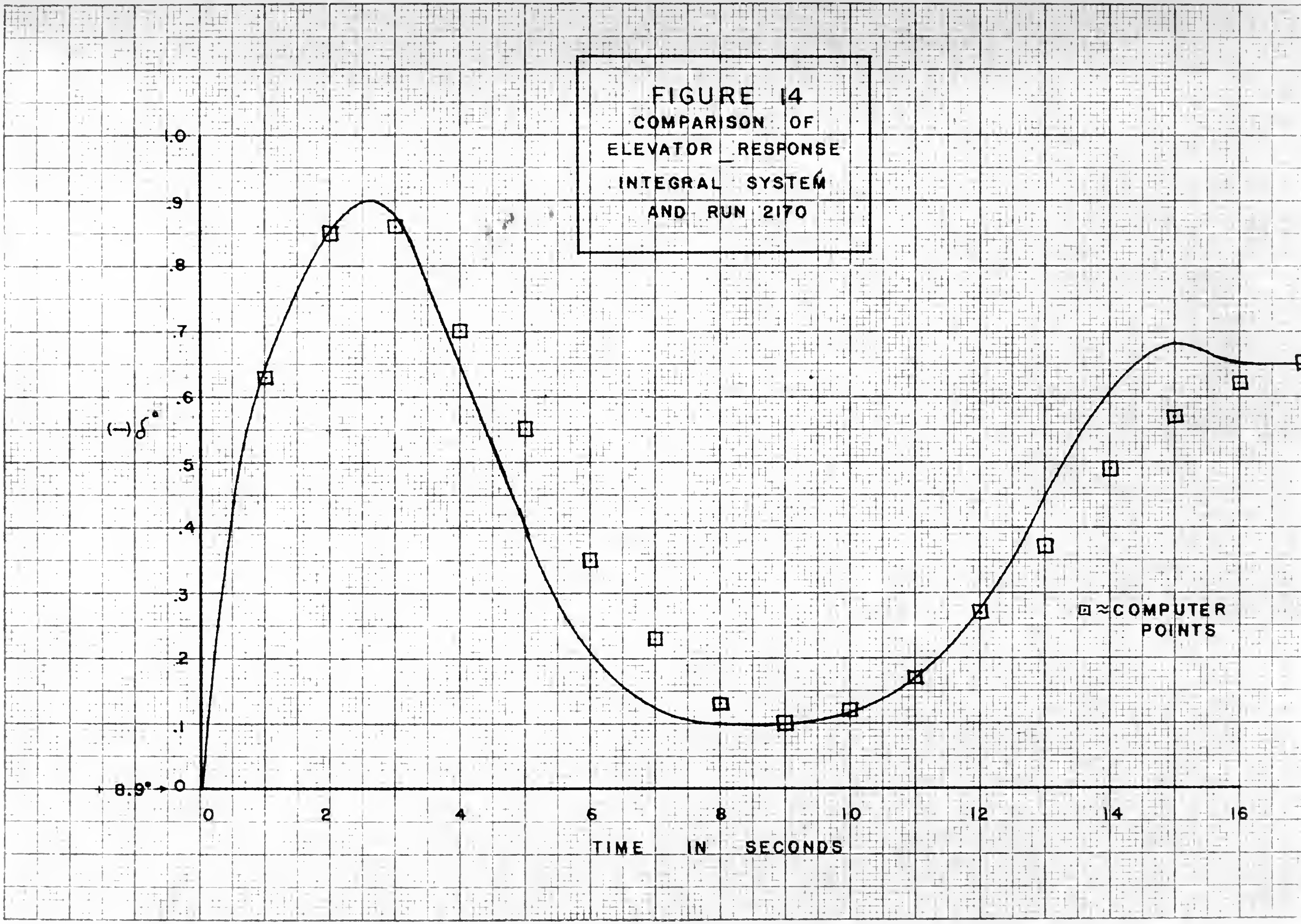


FIGURE 14
COMPARISON OF
ELEVATOR RESPONSE
INTEGRAL SYSTEM
AND RUN 2170
(—) δ°
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
+8.9° → 0
0
2
4
6
8
10
12
14
16
TIME IN SECONDS
⊡ ≈ COMPUTER POINTS

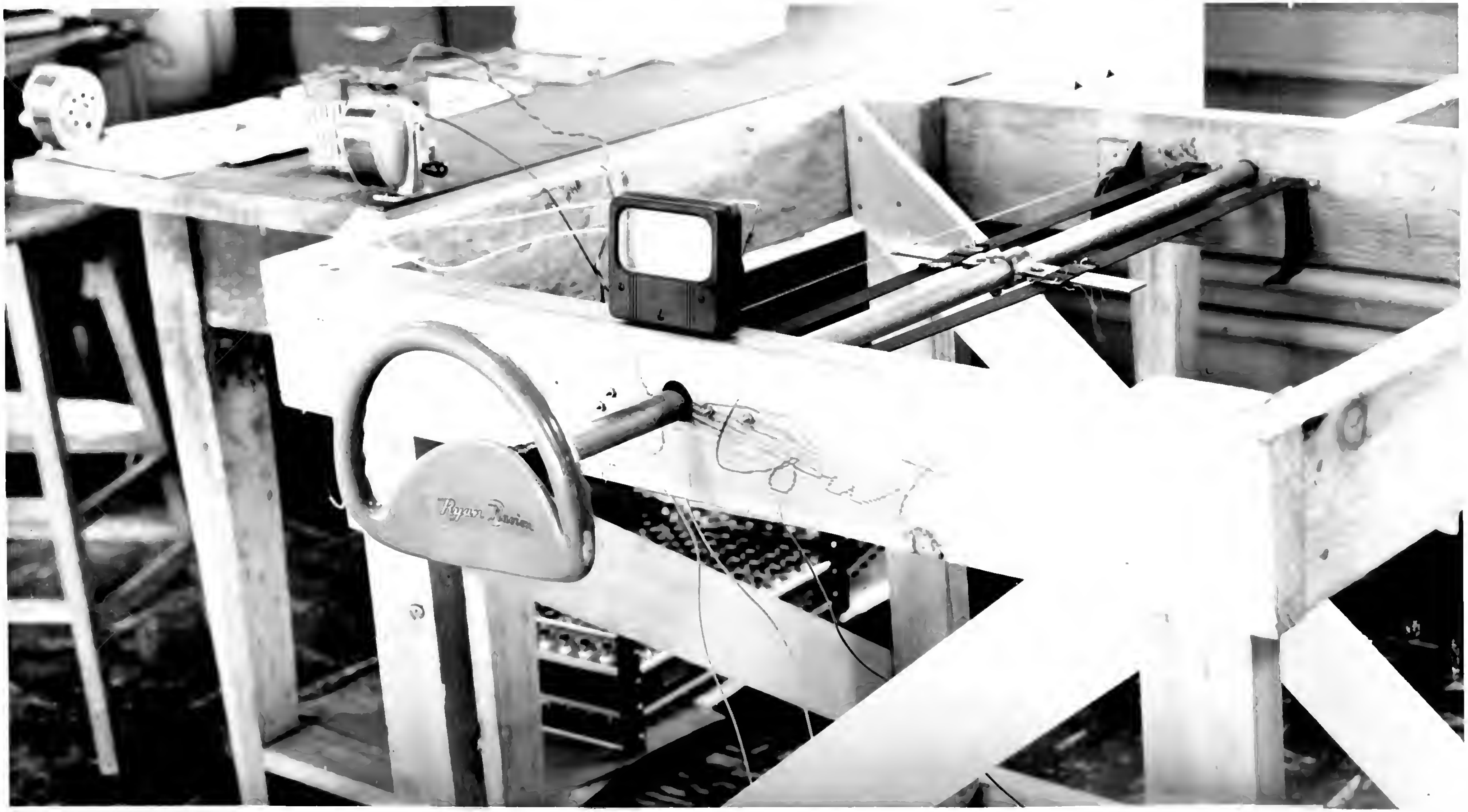


FIGURE 15 - ELEVATOR CONTROL MOCK-UP

FIGURE 16 - ELEVATOR CONTROL MOCK-UP

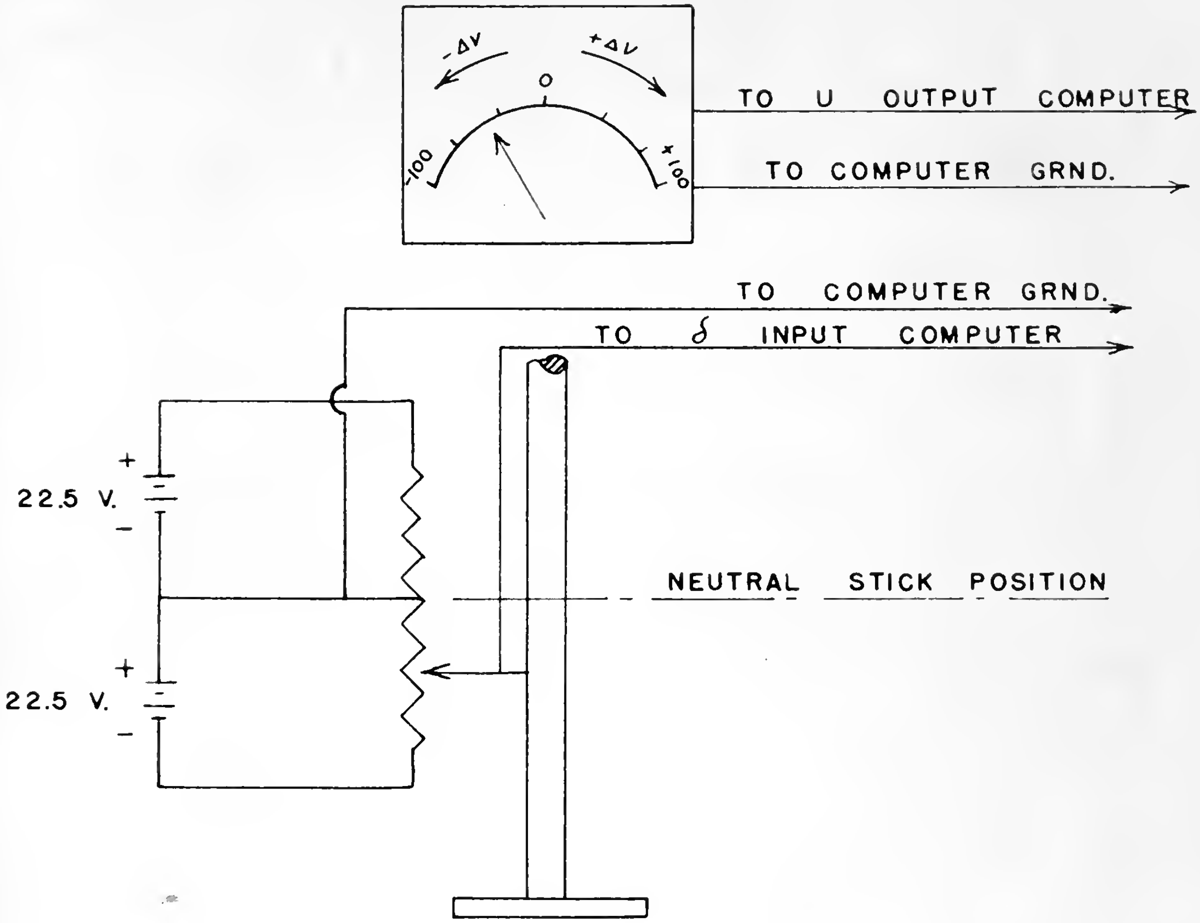


FIGURE 17

SCHEMATIC OF MOCK-UP ELECTRICAL SYSTEM

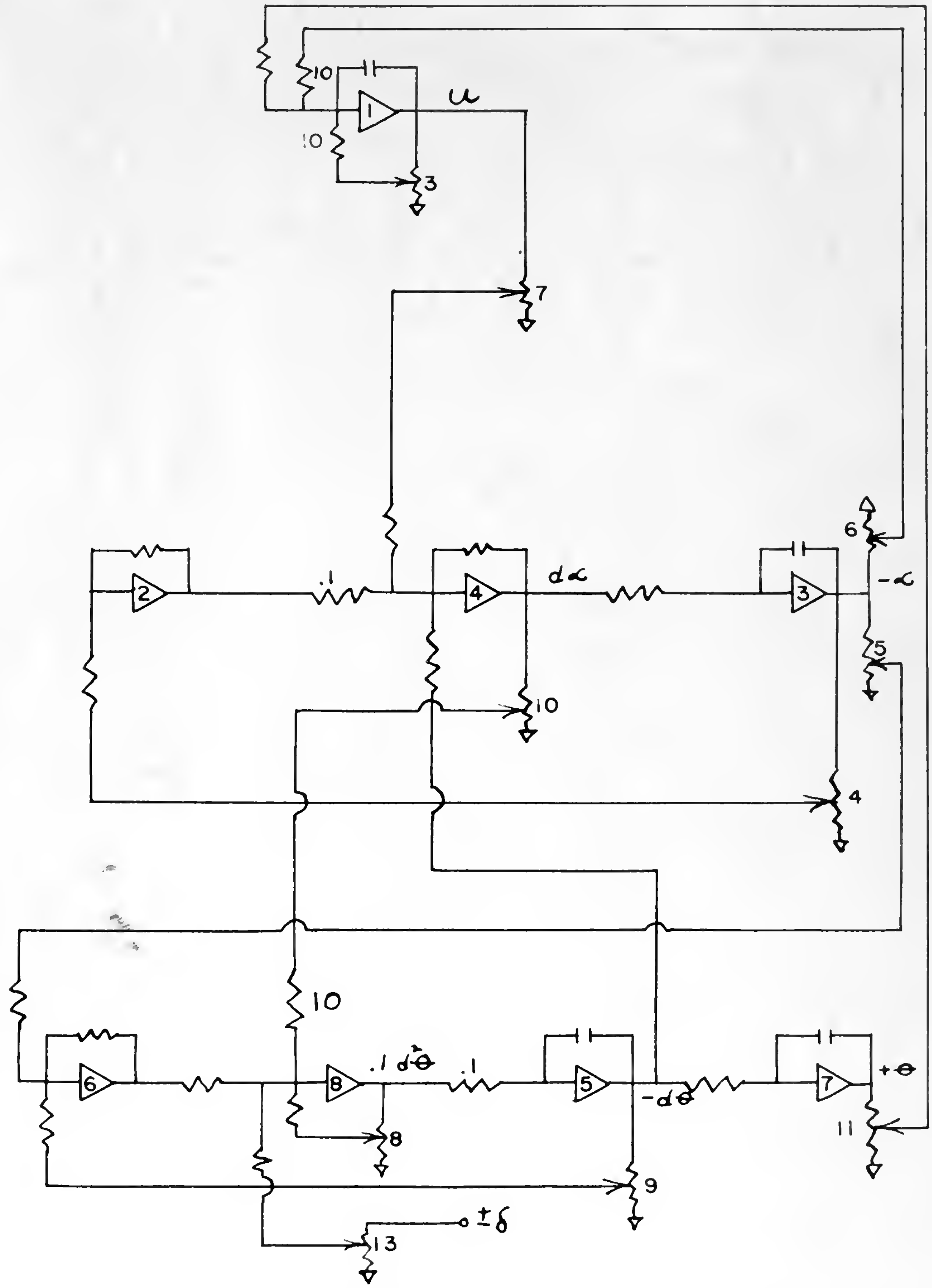


FIGURE 18

ANALOG SCHEMATIC —
AIRCRAFT EQUATIONS OF MOTION

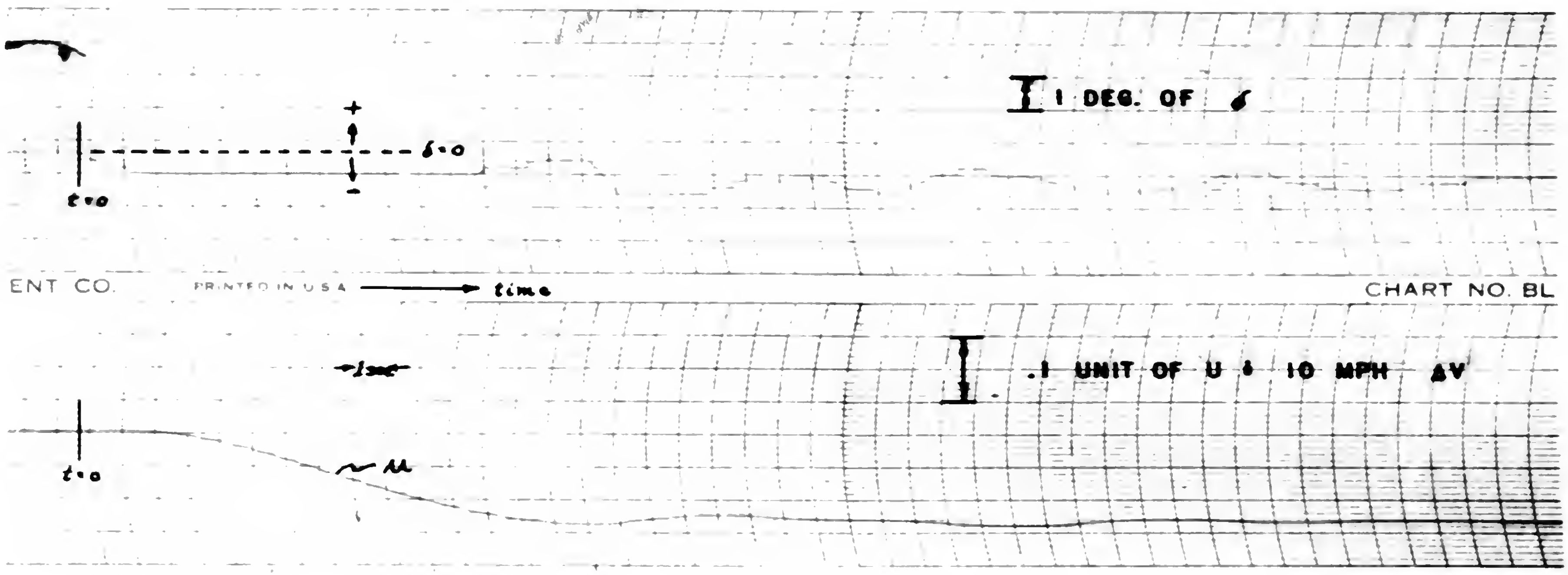


FIGURE 19

13 MPH AIRSPEED CHANGE — COMPUTER

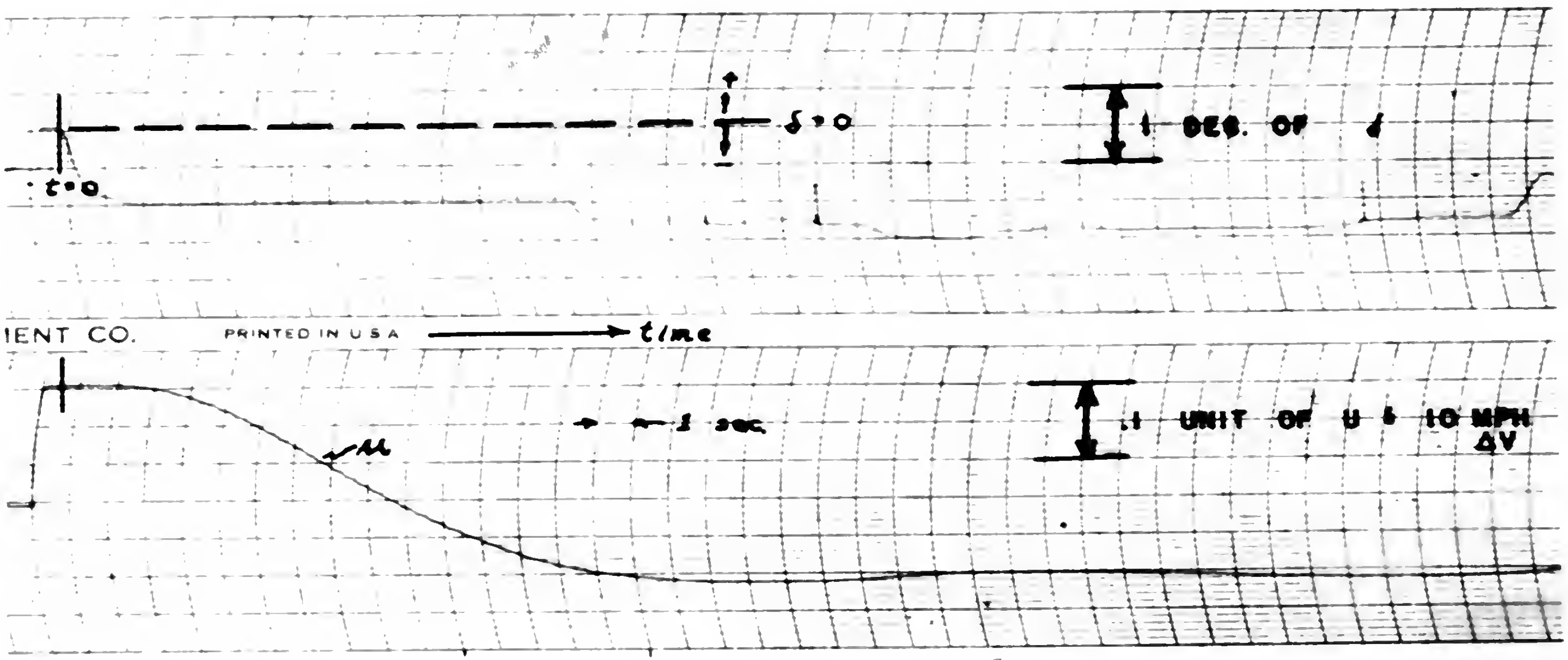


FIGURE 20

26 MPH AIRSPEED CHANGE – COMPUTER

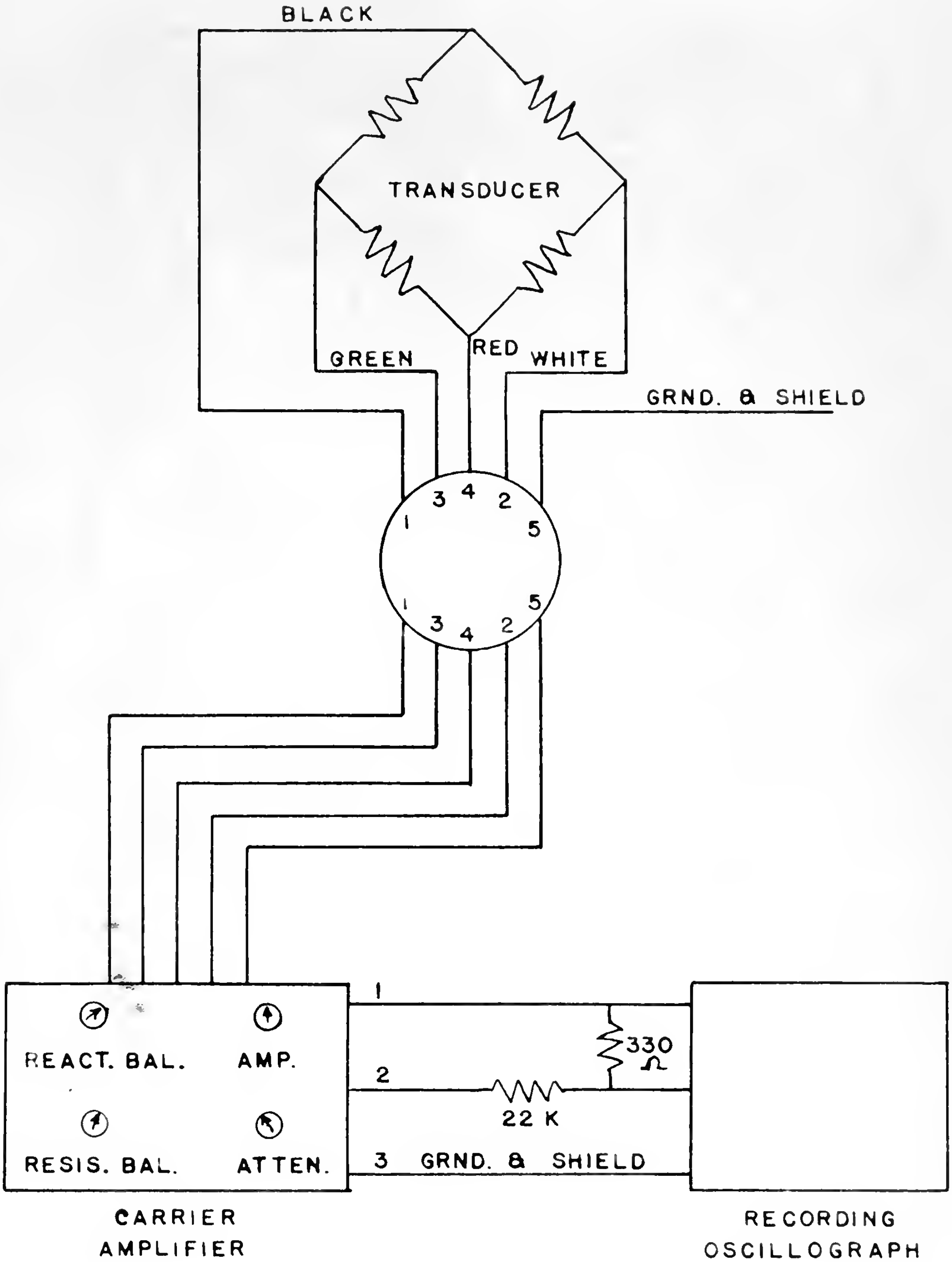


FIGURE 21

SCHEMATIC DIAGRAM OF VELOCITY RECORDING SYSTEM

FIGURE 22

PRESSURE TRANSDUCER INSTALLATION

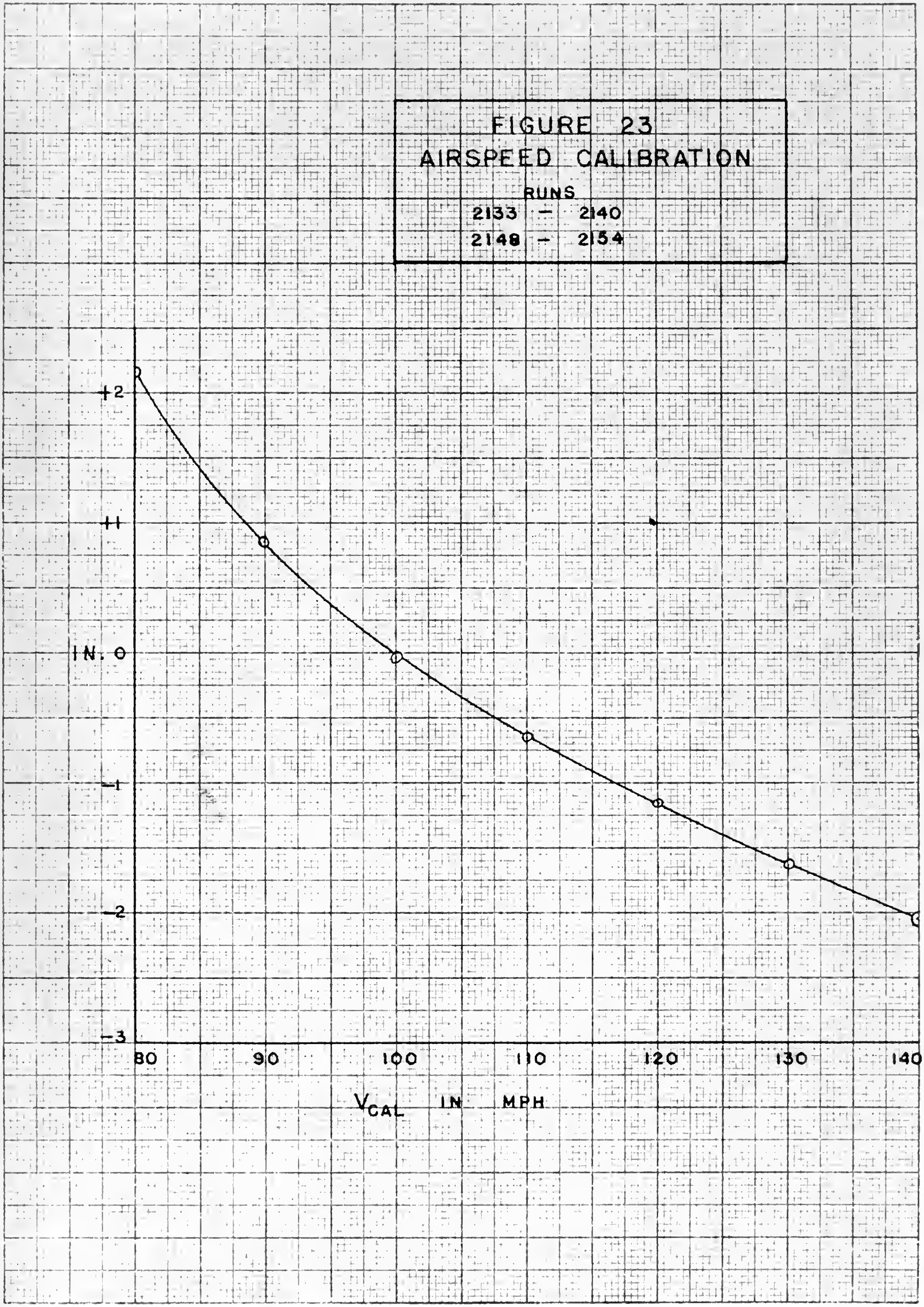


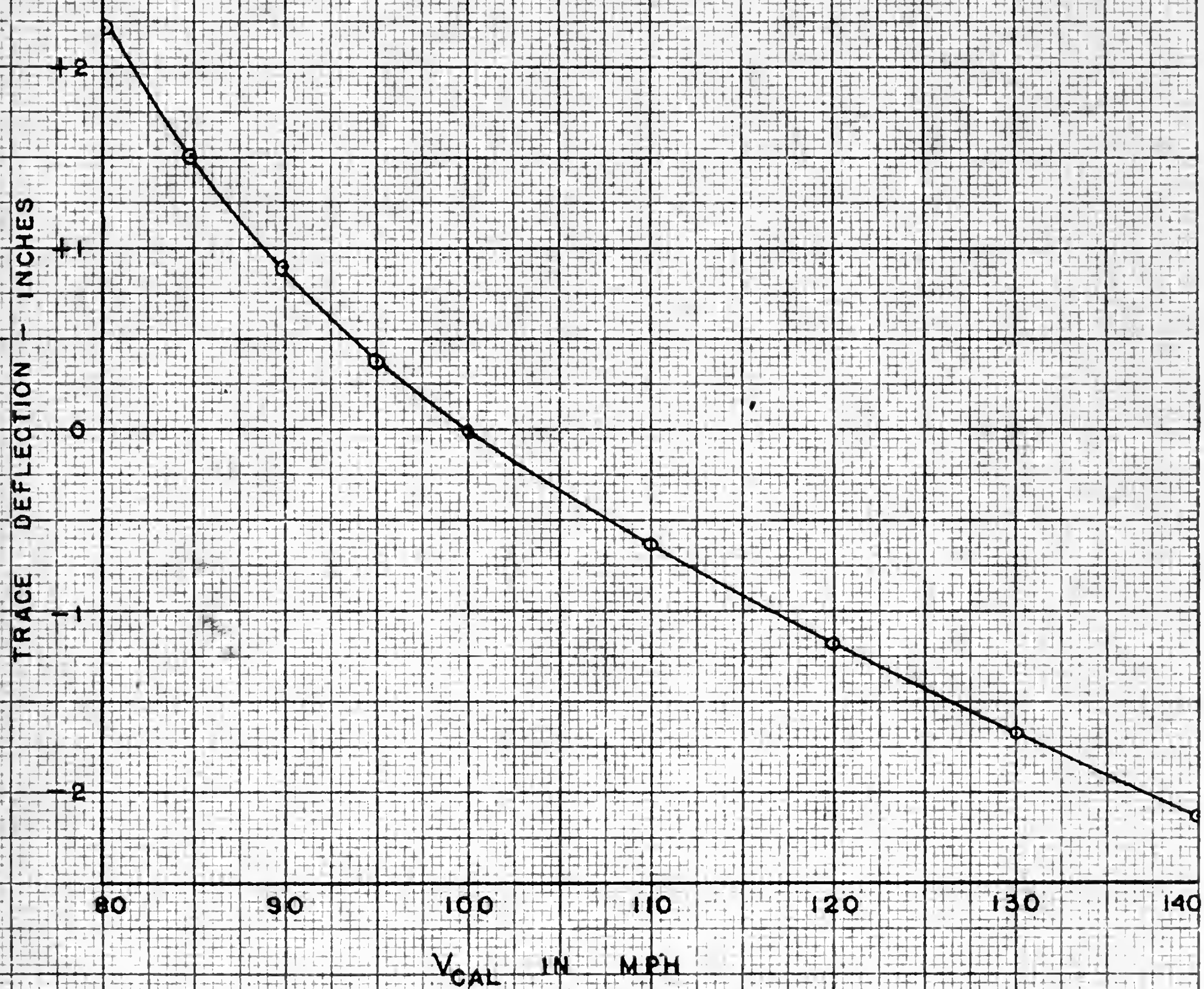
FIGURE 23
AIRSPEED CALIBRATION
RUNS
2133 – 2140
2148 – 2154
+2
+1
IN. 0
-1
-2
-3
80
90
100
110
120
130
140
V_{CAL} IN MPH

FIGURE 24

AIRSPEED CALIBRATION

RUNS

2165 - 2174



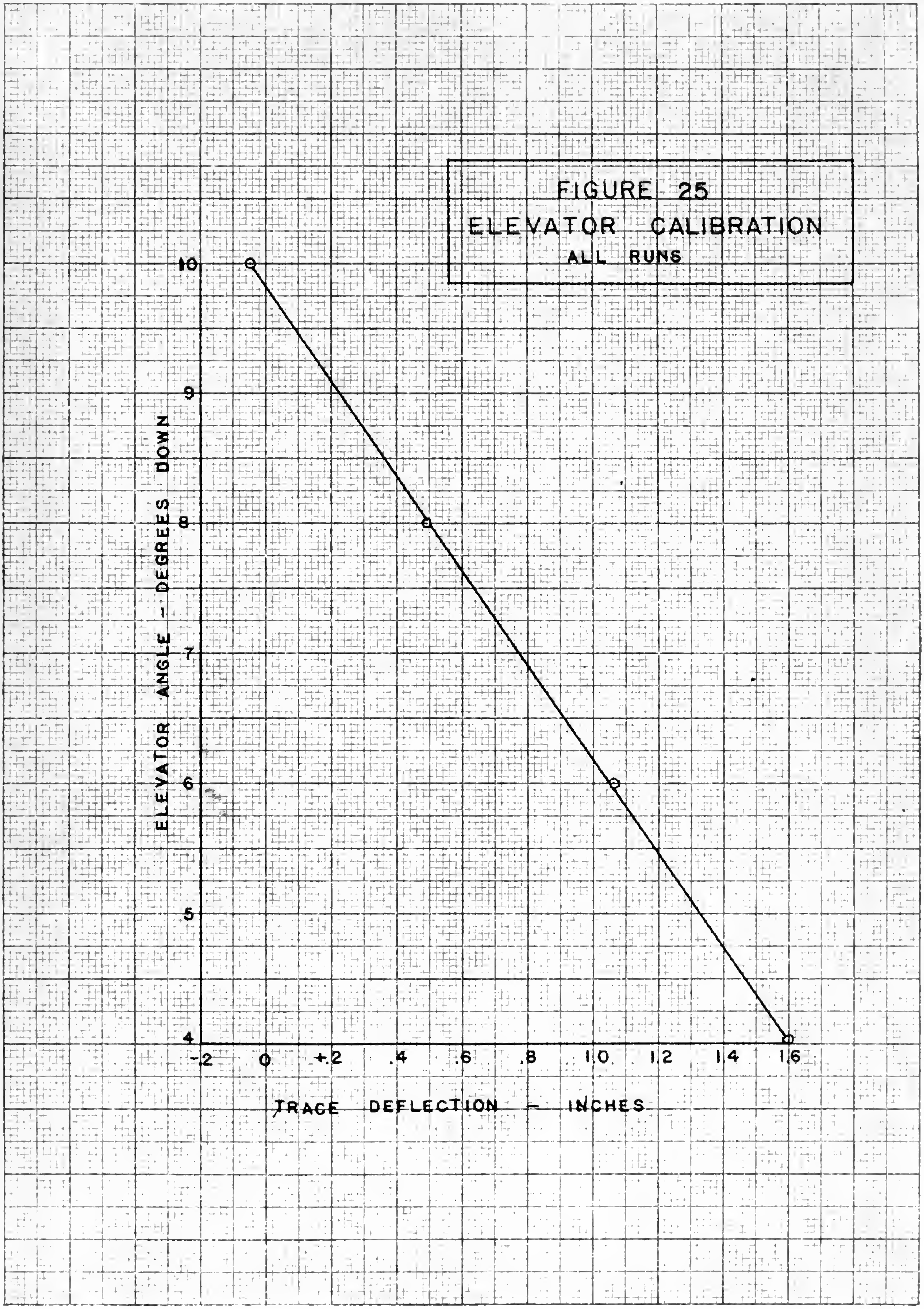


FIGURE 25
ELEVATOR CALIBRATION
ALL RUNS
ELEVATOR ANGLE - DEGREES DOWN
10
9
8
7
6
5
4
-.2
0
+.2
.4
.6
.8
1.0
1.2
1.4
1.6
TRACE DEFLECTION - INCHES

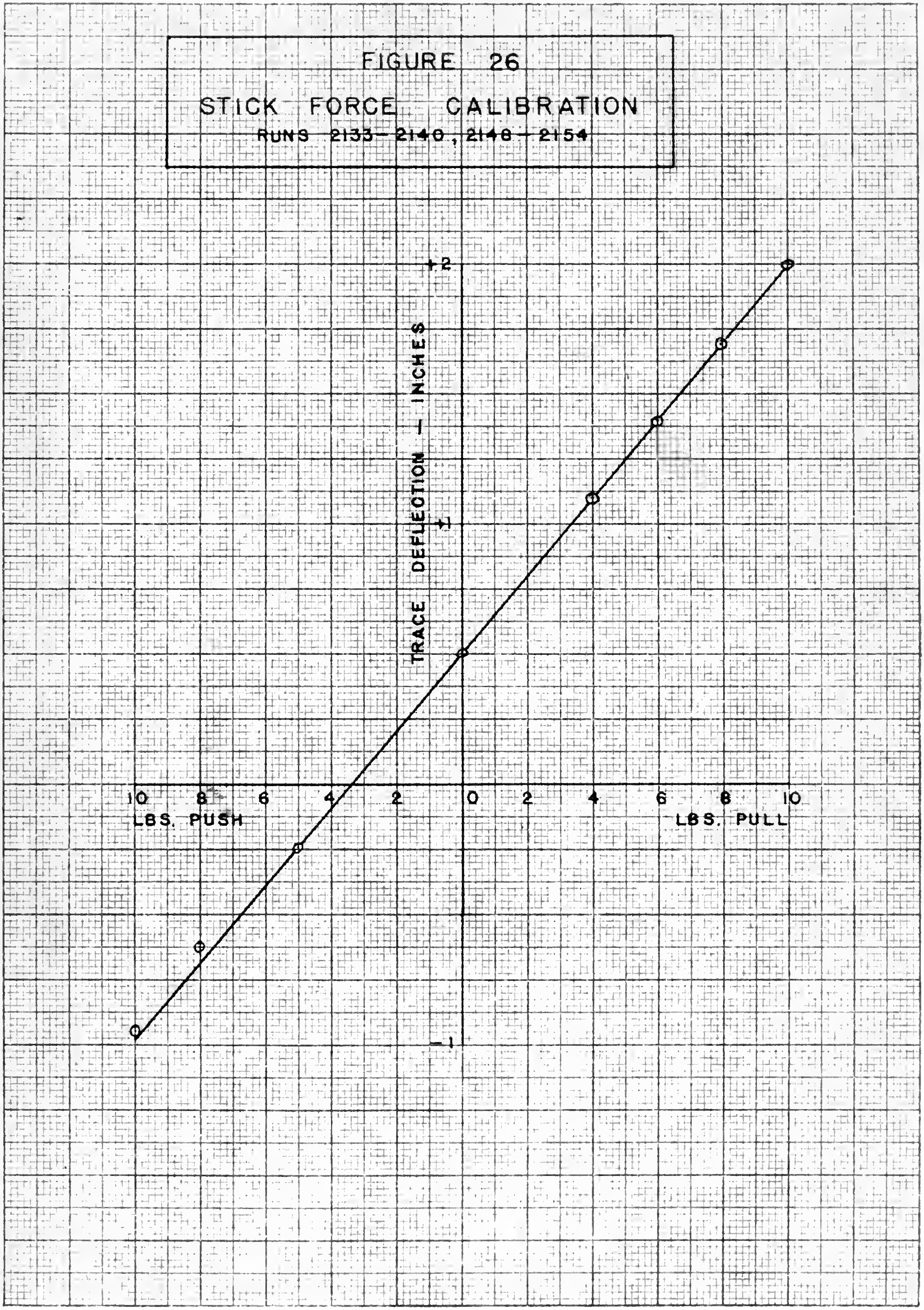


FIGURE 26
STICK FORCE CALIBRATION
RUNS 2133-2140, 2148-2154
+2
+1
-1
TRACE DEFLECTION - INCHES
10 8 6 4 2 0 2 4 6 8 10
LBS. PUSH
LBS. PULL